Study Note—Euclid's *Elements*, Book I, Proposition 48

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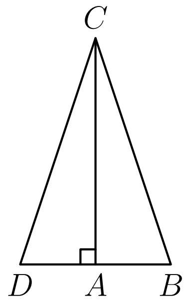
This proposition establishes that if a triangle ABC has the property that the square on the side BC is equal to the sum of the squares on the sides BA and AC, then that triangle is a right-angled triangle, with the right angle being the angle BAC at the vertex A.

In the proof of this result given by Euclid, a right angled triangle ADC is constructed so that the following conditions are satisfied: the side AC is common to both triangles; the angle of the triangle ADC at A is a right angle; the two triangles lie on opposite sides of the side AC; the sides AB and AD of the two triangles are equal in length.



Denoting the square on the side DC by Quad(DC), and adopting analogous notation for the squares on the other sides depicted in the diagram, and applying Proposition 47 of Book I of the *Elements* (which establishes Pythagoras' Theorem), we see that

$$\begin{aligned} \mathrm{Quad}(DC) &= \mathrm{Quad}(DA) + \mathrm{Quad}(AC) \\ &= \mathrm{Quad}(BA) + \mathrm{Quad}(AC) \\ &= \mathrm{Quad}(BC). \end{aligned}$$

Consequently the sides DC and BC are equal in length.

The three sides DC, DA and AC of the triangle ADC are then respectively equal in length to the three sides BC, BA and AC of the triangle ABC. Applying the SSS Congruence Rule (established in Proposition 8 of Book I of the *Elements*), we conclude that the two triangle are congruent to one another, and consequently $\angle BAC = \angle DAC$. But the angle DAC is a right angle. Consequently the angle BAC is a right angle, as required.