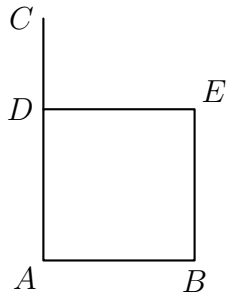
Study Note—Euclid's *Elements*, Book I, Proposition 46

David R. Wilkins

In this proposition, Euclid sets out how to construct a square having as one of its sides a given line segment, thus proving the existence of such squares.

Let AB be the given straight line. A straight line AC is constructed with endpoint at A which is perpendicular to AB which is at least as long as AB. (Proposition 11 of Book I of the *Elements* establishes the that this construction is possible.) A point D is taken on AC located so that $AD = AB$. Then the straight line segment DE is then constructed equal and parallel to AB, with one endpoint at the point D, and lying on the same side of AC as the straight line segment AB. (Proposition 31 of Book I of the *Elements* establishes the that this construction is possible.) Then the points B and E are joined by a straight line segment.



Proposition 33 of Book I of the *Elements* ensures that BE is equal and parallel to AD. Consequently the four sides of the quadrilateral $ABED$ are equal in length.

Proposition 34 of Book I of the *Elements* ensures that

$$\angle BAD = \angle BED \quad \text{and} \quad \angle ADE = \angle ABE.$$

Proposition 29 of Book I of the *Elements* ensures that

$$\angle BAD + \angle ADE = \text{two right angles.}$$

But $\angle BAD$ is a right angle. Consequently $\angle ADE$ is a right angle. It then follows that all four angles BAD, ABE, ADE and BED are right angles. We conclude therefore that $ABED$ is a square, as required.