# Study Note—Euclid's *Elements*, Book I, Proposition 45

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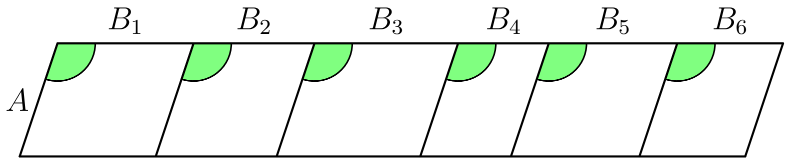
The nature and generality of this proposition can perhaps be made apparent through the use of modern subscript notation. We suppose that we are given a fixed line segment $A$ together with a fixed angle $\theta$. We are also given a rectilineal figure $F$. That figure may be divided as a finite union of triangles $T_1, T_2, \ldots, T_k$ whose interiors are disjoint. The figure $F$ is then equal in area to the sum of the triangles $T_1, T_2, \ldots, T_k$. The preceding proposition, Proposition 44, shows that line segments $B_1, B_2, \ldots, B_k$ can be constructed so that, for each integer $i$ between 1 and $k$, the parallelogram contained by sides equal to $A$ and $B_i$ meeting one another so as to contain an angle equal to $\theta$ is equal in area to the triangle $T_i$. We can then construct a straight line segment $B$ that can be subdivided into $k$ straight line segments which are equal in length to $B_1, B_2, \ldots, B_k$. The parallelogram with containing sides




equal to $A$ and $B$ meeting one another so as to contain an angle equal to $\theta$ is then equal in area to the sum of the parallelograms those containing sides are equal to $A$ and $B_i$ for $i = 1, 2, \ldots, k$, and is therefore equal in area to the sum of the triangles $T_1, T_2, \ldots, T_k$. Consequently the parallelogram with containing sides equal in length to $A$ and $B$ and meeting at angle $\theta$ is equal in area to the given rectilineal figure.