# Study Note—Euclid's *Elements*, Book I, Proposition 44

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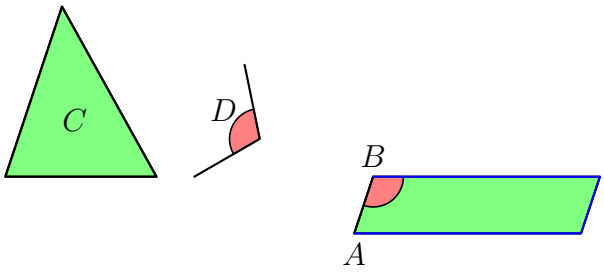
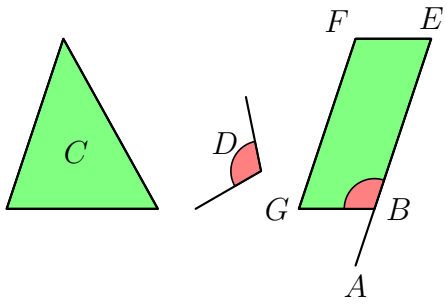
In the configuration considered in this proposition we are given a straight line segment $AB$, a triangle $C$ and an angle $D$. It is required to construct a parallelogram having the given straight line segment $AB$ as one of its sides so as to ensure that the parallelogram is equal in area to the given triangle $C$ and also that the angle of the parallelogram at the vertex $B$ is equal to the given angle $D$.

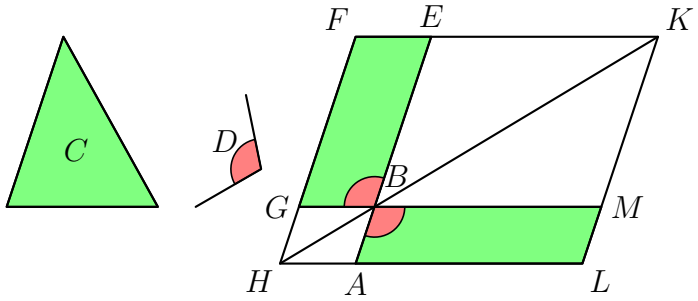



Now, Proposition 42 of Book I of the *Elements* ensures that a parallelogram $BEFG$ can be constructed so as to ensure that the parallelogram is equal in area to the triangle $C$ and also that the angle of the parallelogram at the vertex $B$ is equal to the angle $D$. Moreover this parallelogram can be



positioned so as to ensure that the points $A$, $B$ and $E$ are collinear, with the point $B$ lying between the points $A$ and $E$.

The parallelogram $ABGH$ can then be completed. The points $A$, $E$, $F$ and $H$ are then the vertices of a parallelogram. It then follows from Proposition 29 of Book I of the *Elements* that the angles of this parallelogram at the vertices $F$ and $H$ add up to two right angles. Consequently, if we join the points $H$ and $B$ by a straight line, the angles $BHG$ and $GFE$ add up to less than two right angles. Consequently it follows, on applying the Fifth Postulate, that the rays from the points $F$ and $H$ passing through the points $E$ and $B$ respectively will intersect at some point $K$. We then construct the point $L$ so as to complete the parallelogram $FHLK$, and we produce the line segment $GB$ in a straight line beyond $B$ to the point $M$ lying on the line $KL$, and we produce the line segment $HB$ beyond $B$ to the point $K$.




Now Proposition 43 of Book I of the *Elements* ensures that the parallelograms $BEFG$ and $ABML$ are equal in area. Consequently the parallelogram $ABML$ and the triangle $C$ are both equal in area to the parallelogram $BEFG$. The parallelogram $ABML$ and the triangle $C$ must therefore be equal to one another in area.

Also Proposition 15 of Book I of the *Elements* ensures that the vertically-opposite angles $ABM$ and $GBE$ are equal to one another. The angle $ABM$ and the given angle $D$ are therefore both equal to the angle $GBE$. It follows that the angle $ABM$ must be equal to the angle $D$. We conclude from this that the parallelogram $ABML$ has been constructed so as to satisfy all the properties specified in the statement of the proposition.