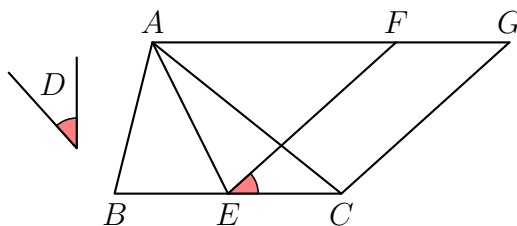# Study Note—Euclid's *Elements*, Book I, Proposition 42

David R. Wilkins

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In the configuration considered in this proposition we are given a triangle ABC and an angle D. It is required to construct a parallelogram equal in area to the triangle ABC, where one of the angles of the constructed parallelogram is equal to the given angle D.

To perform the construction, the base BC of the triangle is bisected at the point E, and the points A and E are joined by a straight line. A ray is then constructed, with its endpoint at the point E, which is directed into the side of the line BC containing the point A so as to make an angle with EC equal to the given angle D. (Proposition 23 of Book I of the *Elements* enables such a construction.) A straight line segment parallel to the base BC of the triangle is then constructed with one endpoint at the point A and the other endpoint F located on the ray previously constructed through the point E so as to make an angle equal to D with the line EC. (Proposition 31 of Book I of the *Elements* enables the construction of the line AF.) The parallelogram $AECG$ is then completed.

Now Proposition 41 of Book I of the *Elements* ensures that the parallelogram $FECG$ is in area double the triangle AEC. Also Proposition 38 of Book I of the *Elements* ensures that the triangles ABE and AEC are equal in area. Consequently the triangle ABC is in area double the triangle AEC. We have now shown that the triangle ABC and the parallelogram $FECG$ are each in area double the triangle AEC. They are therefore equal to one
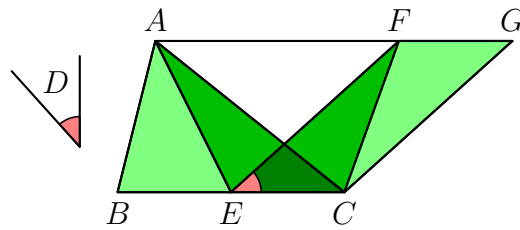
another in area. Moreover the angle FEC of that parallelogram is equal to the given angle D. Thus the parallelogram $FECG$ constructed in this fashion satisfies the requirements set out in the proposition.