Study Note—Euclid's *Elements*, Book I, Proposition 36

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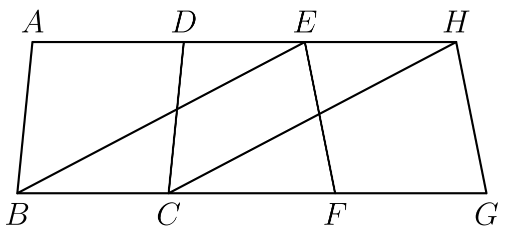
For this proposition we are given two parallelograms $ABCD$ and $EFGH$ with equal bases BC and FG which are in the same parallels, so that the points A, D, E and H are collinear, the points B, C, F and G are collinear and the two lines upon which these points lie are parallel to one another. We are required to prove that these two parallelograms are equal in area.

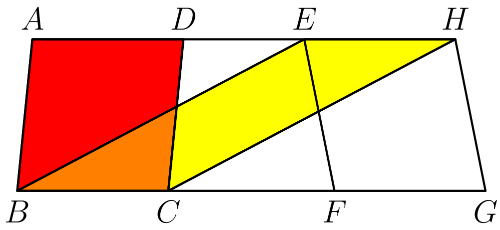


First Euclid uses Proposition I.34 and Common Notion 1 to establish that $BC = EH$.

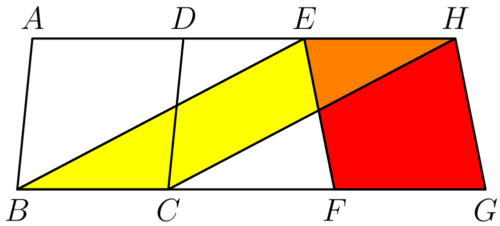
In detail, we are given that $BC = FG$, and Proposition I.34 establishes that $EH = FG$. Consequently BC and EH, being both equal to FG, must be equal to one another.

Applying Proposition I.33, we conclude from this that $EBCH$ is a parallelogram.

We have now established that $ABCD$ and $EBCH$ are parallelograms with the same base BC and between the same parallels. Proposition I.35 now ensures that $ABCD$ and $EBCH$ are equal in area.



An analogous argument establishes that $EFGH$ and $EBCH$ are parallelograms on the same base EH and between the same parallels, and consequently $EFGH$ and $EBCH$ are equal in area.



It now follows, on applying Common Notion 1 that the parallelograms $ABCD$ and $EFGH$, being both equal in area to the parallelogram $EBCH$, must be equal in area to each other, as required.