Study Note—Euclid's *Elements*, Book I, Proposition 34

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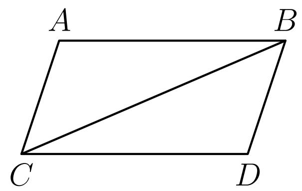
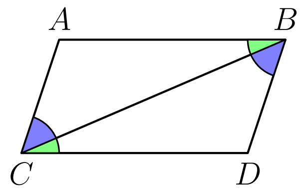
For this proposition we are given a parallelogram $ABDC$ for which the sides AB and CD are parallel to one another and the sides AC and BD are also parallel to one another. We must show that $AB = CD$, $AC = BD$, $\angle BAC = \angle CDB$ and $\angle ACD = \angle DBA$. Once this has been shown, an immediate application of Proposition I.4 establishes that the triangles ABC and DCB are equal to one another in area.



Now, because AB and CD are parallel to one another, the angles of the top left and bottom right triangles at B and C respectively, being alternate angles, must be equal to one another. (This follows immediately on applying Proposition I.29.) Thus $\angle ABC = \angle DCB$.

Similarly, because AC and BD are parallel to one another, the angles of the top left and bottom right triangles at C and B respectively, being alternate angles, must be equal to one another. Thus $\angle BCA = \angle CBD$.



Applying the ASA Congruence Rule (established as part of Proposition I.26) with respect to the side BC common to both triangles, we conclude that the two triangles are congruent to one another, and consequently $AB = DC$ and $AC = DB$, and moreover $\angle BAC = \angle CDA$.

Now the sum of the angles of the two triangles at the common vertex C is equal to sum of the angles of the two triangles at the common vertex B. Consequently $\angle ACD = \angle DBA$. The required results have therefore been established.