Study Note—Euclid's *Elements*, Book I, Proposition 33

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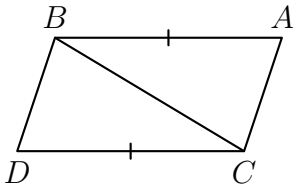
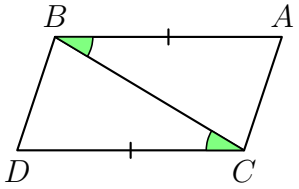
For this proposition we are given a quadrilateral $ABDC$ for which the sides AB and CD are equal to one another in length and are also parallel to one another. We must show that the sides AC and BD are equal to one another in length and are also parallel to one another.



Now, because the lines AB and CD are parallel to one another, it follows (on applying Proposition I.29) that the alternate angles ABC and BCD are equal to one another.



Consequently the two sides AB, BC and the included angle ABC are respectively equal to the two sides DC, CB and the included angle DBC. Applying the SAS Congruence Rule (Proposition I.4) to the triangles ABC and DCB at vertices B and C respectively, we conclude that these two triangles are congruent, and consequently $AC = BD$ and $\angle ACB = \angle DBC$. The equality of these two latter angles then ensures (by Proposition I.27) that the lines AC and BD are parallel, as required.