Study Note—Euclid's *Elements*, Book I, Proposition 29

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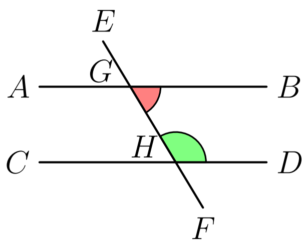
Proposition 29 is the first proposition in Book I of Euclid's *Elements of Geometry* to require the Fifth Postulate for its proof. We recall the statement of the Fifth Postulate:—

> **Fifth Postulate.** *If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles.*

Let a straight line EF intersect two parallel straight lines AB and CD at points G and H respectively, as depicted in the following figure.



If the sum of the internal angles BGH and GHD were less than two right angles, then the Fifth Postulate would ensure that the straight lines AB and CD, if produced sufficiently far in straight lines beyond B and D respectively, would intersect at some point, contradicting the requirement that these straight lines are parallel to one another. Consequently the sum of the internal angles BGH and GHD cannot be less than two right angles.

Considering the angles on the other side of the line EF, we conclude, for similar reasons, that the sum of the internal angles AGH and GHC cannot be less than two right angles. It follows from this (applying Proposition I.13) that the sum of the internal angles BGH and GHD cannot be greater than two right angles.

Therefore, if the lines AB and CD are parallel to one another, then the internal angles BGH and GHD must be equal to two right angles.

It follows that if the lines AB and CD are parallel to one another then

$$\angle AGH + \angle BGH = \text{two right angles} = \angle GHD + \angle BGH,$$

and therefore the alternate angles AGH and GHD are equal to one another. This result is often referred to as the *Alternate Angles Theorem.* The converse of this theorm was established by Euclid in Proposition 27 of Book I of the *Elements of Geometry*

Also if the lines AB and CD are parallel to one another then

$$\angle EGB + \angle BGH = \text{two right angles} = \angle GHD + \angle BGH,$$

and therefore the corresponding angles EGB and GHD are equal to one another. This result is often referred to as the *Corresponding Angles Theorem.* The converse of this theorm was established by Euclid in Proposition 28 of Book I of the *Elements of Geometry*