Study Note—Euclid's *Elements*, Book I, Proposition 28

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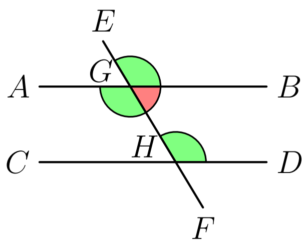
Let a straight line EF intersect straight lines AB and CD at points G and H respectively, as depicted in the figure. The angle EGB is an *exterior* angle with regard to this configuration. Also the angles BGH and GHD are *interior angles on the same side*, and the angle GHD is the *interior and opposite angle on the same side as* the external angle EGB. Furthermore, the angles EGB and GHD are often referred to as *corresponding angles* with respect to the configuration under discussion.



Suppose then that the corresponding angles EGB and GHD are equal, or, in other words, suppose that the exterior angle EGH is equal to the interior and opposite angle GHD. Now the angle EGB is equal to the vertically opposite angle AGH (Proposition I.15). Consequently

$$\angle AGH = \angle EGB = \angle GHD.$$

The equality of the alternate angles AGH and GHD then ensures (by Proposition I.27) that the lines AB and CD are parallel to one another. This result is the converse of the *Corresponding Angles Theorem* that is a consequence of the results established by Euclid in Proposition 29 of Book I of the *Elements of Geometry*.

The other result established by Euclid in Proposition 28 asserts that (with the same diagram) if the two internal angles BGH and GHD on the same side add up to two right angles then the lines AB and CD are parallel. In this situation it follows (using Proposition I.13) that

$$\angle GHD + \angle BGH = \text{two right angles} = \angle AGH + \angle BGH,$$

and consequently (applying the Third Common Notion) $\angle GHD = \angle AGH$. The equality of the alternate angles AGH and GHD then ensures (by Proposition I.27) that the lines AB and CD are parallel to one another.