Study Note—Euclid's *Elements*, Book I, Proposition 27

David R. Wilkins

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Let a straight line intersect straight lines AB and CD at points E and F respectively, as depicted in the figure. The angles AEF and EFD are said to be alternate angles (or alternate interior angles) with regard to this configuration.



Suppose that the straight lines AB and CD, when produced in straight lines beyond B and D respectively, were to meet at some point G. Then the angle AEF would be an external angle of the triangle EFG, and consequently (by Proposition I.16) the angle AEF would be greater than the internal and opposite (or remote) angle EFD of the triangle EFG.



Alternatively suppose that the straight lines AB and CD, when produced in straight lines beyond A and C respectively, were to meet at some point H. Then the angle EFD would be an external angle of the triangle EFH, and consequently (by Proposition I.16) the angle EFD would be greater than the internal and opposite (or remote) angle AEF of the triangle EFH.



It follows immediately from these observations that if the alternate angles AEF and EFD are equal to one another, then the lines AB and CD must be parallel to one another. This result is the converse of the *Alternate* Angles Theorem that is a consequence of the results established by Euclid in Proposition 29 of Book I of the *Elements of Geometry*.