Study Note—Euclid's *Elements*, Book I, Proposition 23

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In the configuration considered in this proposition we are given a straight line segment AB and a rectilineal angle C, formed at a point that is an endpoint of two distinct straight line segments. The proposition presents a construction, attributed by Eudemus of Rhodes to the ancient Greek mathematician Oenopides, for constructing a rectilineal angle at the point A that is equal to the given angle C.



To carry out the construction, one first takes points D and E, one on each of the two line segments that meet at the point C to form the angle there. One should ensure that CE is shorter than AB. Then the points Dand E are joined by a straight line segment.



Now Proposition 20 of Book I of the *Elements* then ensures that the sum of any two sides of the triangle CDE is greater than the remaining side. It then follows, on applying Proposition 22 of the same book, that a triangle AFG can be constructed, with the vertex G located on the straight line segment AB between A and B, so that

$$AG = CE$$
, $AF = CD$ and $FG = DE$.

It then follows, on applying the SSS Congruence Rule established in Proposition 8 of Book I of the *Elements*, that the triangles AFG and CDE are congruent to one another, and consequently the angle FAG is equal to the given angle DCE at the point C, as required.

