Study Note—Euclid's *Elements*, Book I, Proposition 18

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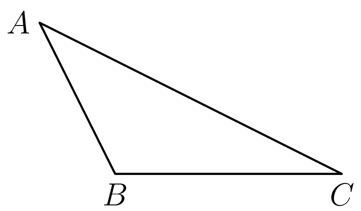
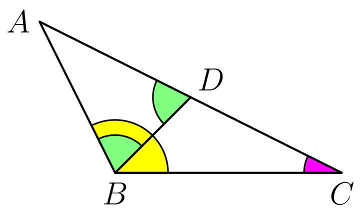
To establish this proposition we must show that, if we are given a triangle with two sides of unequal length, the angle subtended by the longer of the two sides is larger than that subtended by the shorter side.

Thus let ABC be a triangle in which the side AC is longer than AB.



Then let D be the point on the side AC for which $AD = AB$, and join the points B and D by a straight line segment.



Now the angle ADB (coloured green) is an external angle of the triangle DBC, and the angle DCB (coloured magenta) is one of the internal and opposite angles of that triangle. It follows from Proposition 16 of Book I of Euclid's *Elements of Geometry* that

$$\angle ADB > \angle DCB = \angle ACB.$$

Also the triangle ABD is an isosceles triangle with equal sides AB and AD. It follows from Proposition 5 of Book I of Euclid's *Elements of Geometry* that the angles (coloured green) subtended by the equal sides are themselves equal to one another, and therefore

$$\angle ADB = \angle ABD.$$

Also the point D lies in the interior of the angle ABC (coloured yellow) and therefore

$$\angle ABC > \angle ABD.$$

Putting these results together, we conclude that

$$\angle ABC > \angle ABD = \angle ADB > \angle ACB.$$

Thus the angle ABC subtended by the longer side AC of the triangle ABC is greater than the angle ACB subtended by the shorter side AB, as required.