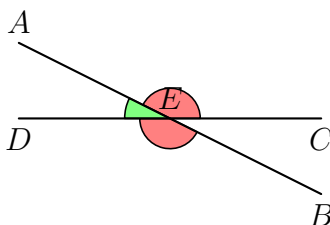


Study Note—Euclid’s *Elements*, Book I, Proposition 15

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This proposition establishes that vertically-opposite angles are equal to one another. Where two straight lines cross one another, two pairs of *vertically-opposite* rectilinear angles are formed. Two rectilinear angles formed at a point E are vertically-opposite if any straight line passing through the point E and also through at least one point in the interior of one of the angles is, with the exception of the point E , contained wholly within the interiors of the angles in question. Thus, in the diagram below, the angles CEA and DEB are vertically-opposite, and the angles BEC and AED are vertically opposite.



Now, applying Proposition 13 of Book I of the *Elements*, we see that

$$\angle CEA + \angle AED = \text{two right angles}$$

and

$$\angle DEB + \angle AED = \text{two right angles}.$$

Consequently the First Common Notion ensures that

$$\angle CEA + \angle AED = \angle DEB + \angle AED.$$

Subtracting the angle AED from both sides, the Third Common Notion then ensures that

$$\angle CEA = \angle DEB.$$

Thus the vertically-opposite angles CEA and DEB are equal to one another. Similarly the vertically-opposite angles AED and BEC are equal to one another, as required.