Study Note—Euclid's *Elements*, Book I, Proposition 14

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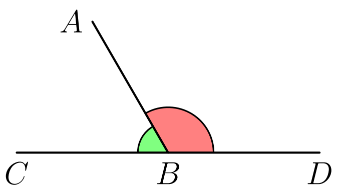
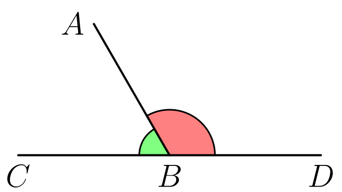
The configuration considered in this proposition may be described as follows. Points C and D lie on opposite sides of a line AB and are situated so as to ensure that, when these points are joined by straight lines to the point B, the sum of the angles CBA and DBA is equal to two right angles. In order to prove the proposition, one must prove that, under these assumptions, the points C, B and D are collinear.

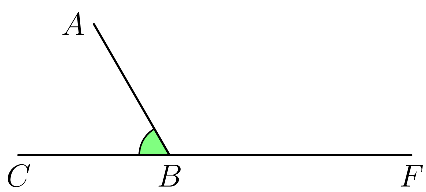


We can justify this result by the following argument which, in its logical structure, departs to some extent from the argument presented by Euclid.

Suppose that the straight line segment CB is produced in a straight line beyond B to some point F. It then follows from Proposition 13 of Book I of the *Elements* that

$$\angle ABC + \angle ABF = \text{two right angles.}$$





Now

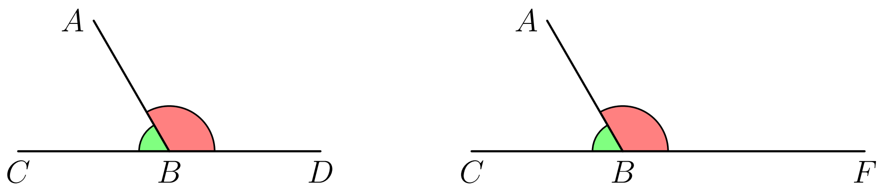
$$\angle ABC + \angle ABD = \text{two right angles,}$$

by assumption. Consequently, applying the First Common Notion, we conclude that

$$\angle ABC + \angle ABF = \angle ABC + \angle ABD.$$

Subtracting the angle ABC from both sides, it follows from the Third Common Notion that

$$\angle ABF = \angle ABD.$$



Now the points D and F both lie on the same side of the line AB. It follows from the equality of the angles ABF and ABD that either the points D and F coincide or else the points B, D and F are collinear. But the points C, B and F are collinear. Consequently the points C and D must both lie on the unique straight line that passes through the points B and F, and consequently the points C, B and D must be collinear, as required.