Study Note—Euclid's *Elements*, Book I, Proposition 13

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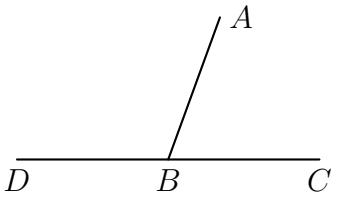
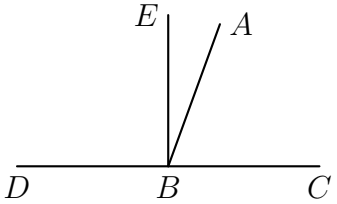
The configuration considered in this proposition may be described as follows. Points C and D are joined by a straight line, a point B is taken on that straight line between the points C and D, and the point B is joined by a straight line to some point A which does not lie on the line. The proposition asserts that, in this configuration, the sum of the angles CBA and ABD is equal to two right angles.



In order the prove the result, a line BE perpendicular to CD is erected at the point B, joining the point B to a point E that lies on the same side of the line CD as the point A. (The possibility of erecting such a perpendicular line was established by Proposition 12 of Book I of the *Elements*.)



Repeatedly applying the Second Common Notion, and considering all right angles to be equal to one another, as asserted in the Fourth Postulate, we find that, in the case where the point A is located in the interior of the

angle EBC,

$$\begin{aligned} \angle DBA + \angle ABC &= \angle DBE + \angle EBA + \angle ABC \\ &= \angle DBE + \angle EBC \\ &= \text{two right angles,} \end{aligned}$$

as required.