Study Note—Euclid's *Elements*, Book I, Proposition 9

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Proposition 9 of Book I of Euclid's *Elements of Geometry* establishes the feasibility of bisecting a rectilineal angle.

Euclid assumes, from the earlier propositions of Book I of the Elements, the following propositions: Proposition 1, which establishes the possibility of constructing an equilateral triangle with a given line segment as one of its sides, Proposition 3, which establishes that, given two line segments of unequal length, a part equal to the shorter can be cut off from the longer, and Proposition 8, which establishes the SSS Congruence Rule. It follows that the geometrical construction of Proposition 9 and its justification could not be introduced at an earlier point in the development of the theory presented in the first book of the *Elements of Geometry*.

We are required to bisect an angle BAC. Applying Proposition I.3, we can find points D and E on AB and AC which are equidistant from the point A. An equilateral triangle DEF is then constructed with the line segment DE joining the points D and E as one of its sides, and with the vertex F on the opposite side of DE to the point A. The points A and F are joined. Then the line segment AF bisects the angle BAC.



To justify this, one notes that the sides DA, AF and DF of the triangle DAF are respectively equal to the sides EA, AF and EF of the triangle EAF. Applying the SSS Congruence Rule (*Elements*, Proposition I.8), we conclude that the angles DAF and EAF are equal to one another, and thus the angle BAC has been bisected by the line segment AF as required.