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The Transportation Problem: The Minimum Cost Method

We now find a basic optimal solution to a transportation problem with 6 suppliers and 5 recipients.

The supply vector is $(9, 14, 5, 16, 7, 9)$ and the demand vector is $(8, 17, 6, 14, 15)$. The components of both the supply vector and the demand vector add up to 60.

The costs are as specified in the following cost matrix:

$$\begin{pmatrix} 12 & 8 & 9 & 4 & 6 \\ 5 & 10 & 8 & 9 & 5 \\ 6 & 4 & 12 & 12 & 4 \\ 5 & 7 & 12 & 10 & 8 \\ 4 & 6 & 8 & 10 & 12 \\ 7 & 3 & 7 & 12 & 8 \end{pmatrix}.$$

The Transportation Problem: The Minimum Cost Method (continued)

We fill in the row sums (or supplies), the column sums (or demands) and the costs $c_{i,j}$ for the given problem. The resultant tableau looks as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 ?	8 ?	9 ?	4 ?	6 ?	9
2	5 ?	10 ?	8 ?	9 ?	5 ?	14
3	6 ?	4 ?	12 ?	12 ?	4 ?	5
4	5 ?	7 ?	12 ?	10 ?	8 ?	16
5	4 ?	6 ?	8 ?	10 ?	12 ?	7
6	7 ?	3 ?	7 ?	12 ?	8 ?	9
d_j	8	17	6	14	15	60

In order to apply the Minimum Cost Method to find an initial basic feasible solution, we identify the cell with the minimum cost associated to it. The minimum cost is 3, and this cost is the value of $c_{i,j}$ only when $i = 6$ and $j = 2$. We therefore set $x_{6,2}$ to be the minimum of the supply s_6 and the demand d_2. Now $s_6 = 9$ and $d_2 = 17$. Therefore we set $x_{6,2} = 9$. Now the values of $x_{i,j}$ in the row $i = 6$ must be non-negative and must sum to 9. It follows that $x_{6,j} = 0$ when $j \neq 2$. We therefore fill in the values of $x_{i,j}$ for $i = 6$, and enter a $\bullet$ symbol in the tableau cell for $(i, j) = (6, 2)$.

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 ?	8 ?	9 ?	4 ?	6 ?	9
2	5 ?	10 ?	8 ?	9 ?	5 ?	14
3	6 ?	4 ?	12 ?	12 ?	4 ?	5
4	5 ?	7 ?	12 ?	10 ?	8 ?	16
5	4 ?	6 ?	8 ?	10 ?	12 ?	7
6	7 0	3 • 9	7 0	12 0	8 0	9
d_j	8	17	6	14	15	60

We next look for the cell or cells of minimum cost amongst those where the value of $x_{i,j}$ is still to be determined. The minimum cost amongst such cells is 4, and $c_{i,j} = 4$ when (i, j) is equal to one of the ordered pairs $(1, 4)$, $(3, 2)$, $(3, 5)$ and $(5, 1)$. Now it makes sense to choose the cell with $c_{i,j} = 4$ for which the value of $x_{i,j}$ will be the maximum possible, since that ensures, in the context of the transformation problem, that the largest amount of the commodity is transported at this cheap price. Thus we chose $(1, 4)$ as our next basis element, set $x_{1,4} = 9$, and set $x_{1,j} = 0$ when $j \neq 0$. The tableau is then as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 / 0	8 / 0	9 / 0	4 • / 9	6 / 0	9
2	5 / ?	10 / ?	8 / ?	9 / ?	5 / ?	14
3	6 / ?	4 / ?	12 / ?	12 / ?	4 / ?	5
4	5 / ?	7 / ?	12 / ?	10 / ?	8 / ?	16
5	4 / ?	6 / ?	8 / ?	10 / ?	12 / ?	7
6	7 / 0	3 • / 9	7 / 0	12 / 0	8 / 0	9
d_j	8	17	6	14	15	60

The minimum cost amongst those cells for which $x_{i,j}$ is still to be determined is still 4. Taking $(i, j) = (5, 1)$ permits the largest possible value of $x_{i,j}$ amongst those cells for which $c_{i,j} = 4$ and $x_{i,j}$ is still to be determined. Therefore we set $x_{5,1}$ to the minimum value of s_5 and d_1, and thus set $x_{5,1} = 7$. In order to achieve a feasible solution we must then take $x_{5,j} = 0$ when $j \neq 1$. Accordingly the first three basis elements determined are

$$(6, 2), \ (1, 4), \ (5, 1),$$

and the tableau at the completion of this stage is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 / 0	8 / 0	9 / 0	4 • / 9	6 / 0	9
2	5 / ?	10 / ?	8 / ?	9 / ?	5 / ?	14
3	6 / ?	4 / ?	12 / ?	12 / ?	4 / ?	5
4	5 / ?	7 / ?	12 / ?	10 / ?	8 / ?	16
5	4 • / 7	6 / 0	8 / 0	10 / 0	12 / 0	7
6	7 / 0	3 • / 9	7 / 0	12 / 0	8 / 0	9
d_j	8	17	6	14	15	60

There are now two remaining cells with cost equal to 4. These are the cells where (i, j) is one of the ordered pairs $(3, 2)$ and $(3, 5)$. At each of these cells, the minimum of s_i and d_j has the value 5. We arbitrarily choose $(3, 2)$ as the next element for the basis, set $x_{3,2} = 5$, and set $x_{3,j} = 0$ for $j \neq 2$. Accordingly the first four basis elements determined are

$$(6, 2), \ (1, 4), \ (5, 1), \ (3, 2)$$

and the tableau at the completion of this stage is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 / 0	8 / 0	9 / 0	4 • / 9	6 / 0	9
2	5 / ?	10 / ?	8 / ?	9 / ?	5 / ?	14
3	6 / 0	4 • / 5	12 / 0	12 / 0	4 / 0	5
4	5 / ?	7 / ?	12 / ?	10 / ?	8 / ?	16
5	4 • / 7	6 / 0	8 / 0	10 / 0	12 / 0	7
6	7 / 0	3 • / 9	7 / 0	12 / 0	8 / 0	9
d_j	8	17	6	14	15	60

We next look for the cell or cells of minimum cost amongst those where the value of $x_{i,j}$ is still to be determined. The minimum cost amongst such cells is 5, and $c_{i,j} = 5$ when (i,j) is equal to one of the ordered pairs $(2,1)$, $(2,5)$, and $(4,1)$. Now taking $(i,j) = (2,1)$ or $(i,j) = (4,1)$ would result in $x_{2,1}$ or $x_{4,1}$ having the value 1, because the numbers in the first column of the body of the tableau must sum to 8. We obtain a larger possible value of $x_{i,j}$ with $(i,j) = (2,5)$, and accordingly we add $(2,5)$ to our basis, set $x_{2,5} = 14$, and set $x_{2,j} = 0$ when $j \neq 5$. Accordingly the first five basis elements determined are

$$(6,2),\ (1,4),\ (5,1),\ (3,2),\ (2,5)$$

and the tableau at the completion of this stage is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 0	8 0	9 0	4 • 9	6 0	9
2	5 0	10 0	8 0	9 0	5 • 14	14
3	6 0	4 • 5	12 0	12 0	4 0	5
4	5 ?	7 ?	12 ?	10 ?	8 ?	16
5	4 • 7	6 0	8 0	10 0	12 0	7
6	7 0	3 • 9	7 0	12 0	8 0	9
d_j	8	17	6	14	15	60

The ordered pair $(4, 1)$ is now the only remaining ordered pair (i, j) for which $c_{i,j} = 5$ and $x_{i,j}$ is still to be determined. We add $(4, 1)$ to our basis. The first column must add up to 8, and accordingly $x_{4,1} = 1$. There are no further cells in the first column for which $x_{i,0}$ is undetermined. Accordingly the first six basis elements determined are

$$(6, 2),\ \ (1, 4),\ \ (5, 1),\ \ (3, 2),\ \ (2, 5),\ \ (4, 1)$$

and the tableau at the completion of this stage is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 / 0	8 / 0	9 / 0	4 • / 9	6 / 0	9
2	5 / 0	10 / 0	8 / 0	9 / 0	5 • / 14	14
3	6 / 0	4 • / 5	12 / 0	12 / 0	4 / 0	5
4	5 • / 1	7 / ?	12 / ?	10 / ?	8 / ?	16
5	4 • / 7	6 / 0	8 / 0	10 / 0	12 / 0	7
6	7 / 0	3 • / 9	7 / 0	12 / 0	8 / 0	9
d_j	8	17	6	14	15	60

The only ordered pairs (i, j) for which $x_{i,j}$ is still to be determined at those with $i = 4$ and $j = 2, 3, 4, 5$. The minimum cost associated with these cells is 7, and corresponds to $j = 2$. Now the second column must sum to 17, and the values of $x_{i,2}$ for $i \neq 4$ sum to 14. Accordingly we must take $x_{4,2} = 3$. (This is compatible with the sum of the row $i = 4$ being equal to 16.) Accordingly the first seven basis elements determined are

$$(6, 2), \ (1, 4), \ (5, 1), \ (3, 2), \ (2, 5), \ (4, 1), \ (4, 2)$$

and the tableau at the completion of this stage is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 / 0	8 / 0	9 / 0	4 • / 9	6 / 0	9
2	5 / 0	10 / 0	8 / 0	9 / 0	5 • / 14	14
3	6 / 0	4 • / 5	12 / 0	12 / 0	4 / 0	5
4	5 • / 1	7 • / 3	12 / ?	10 / ?	8 / ?	16
5	4 • / 7	6 / 0	8 / 0	10 / 0	12 / 0	7
6	7 / 0	3 • / 9	7 / 0	12 / 0	8 / 0	9
d_j	8	17	6	14	15	60

The next lowest cost with undetermined $x_{i,j}$ is 8, and occurs for $(i,j) = (4,5)$. We add $(4,5)$ to the basis. We must take $x_{4,5} = 1$ in order to ensure that $\sum_{i=1}^{6} x_{i,5} = d_5 = 15$. Accordingly the first eight basis elements determined are

$$(6,2),\ (1,4),\ (5,1),\ (3,2),\ (2,5),\ (4,1),\ (4,2)\ (4,5)$$

and the tableau at the completion of this stage is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 / 0	8 / 0	9 / 0	4 • / 9	6 / 0	9
2	5 / 0	10 / 0	8 / 0	9 / 0	5 • / 14	14
3	6 / 0	4 • / 5	12 / 0	12 / 0	4 / 0	5
4	5 • / 1	7 • / 3	12 / ?	10 / ?	8 • / 1	16
5	4 • / 7	6 / 0	8 / 0	10 / 0	12 / 0	7
6	7 / 0	3 • / 9	7 / 0	12 / 0	8 / 0	9
d_j	8	17	6	14	15	60

The next lowest cost with undetermined $x_{i,j}$ is 10, and occurs for $(i,j) = (4,4)$. We add $(4,4)$ to the basis. We must take $x_{4,4} = 5$ in order to ensure that $\sum_{i=1}^{6} x_{i,4} = d_4 = 14$. Accordingly the first nine basis elements determined are

$$(6,2),\ (1,4),\ (5,1),\ (3,2),\ (2,5),\ (4,1),\ (4,2)$$
$$(4,5),\ (4,4)$$

and the tableau at the completion of this stage is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 / 0	8 / 0	9 / 0	4 • / 9	6 / 0	9
2	5 / 0	10 / 0	8 / 0	9 / 0	5 • / 14	14
3	6 / 0	4 • / 5	12 / 0	12 / 0	4 / 0	5
4	5 • / 1	7 • / 3	12 / ?	10 • / 5	8 • / 1	16
5	4 • / 7	6 / 0	8 / 0	10 / 0	12 / 0	7
6	7 / 0	3 • / 9	7 / 0	12 / 0	8 / 0	9
d_j	8	17	6	14	15	60

The Transportation Problem: The Minimum Cost Method (continued)

We complete the table by adding $(4, 3)$ to the basis and setting $x_{4,3} = 6$, so as to ensure that $\sum_{j=1}^{5} x_{4,j} = s_4 = 16$. and $\sum_{i=1}^{6} x_{i,3} = d_3 = 6$. Accordingly the complete basis consists of the ordered pairs

$$(6, 2),\ \ (1, 4),\ \ (5, 1),\ \ (3, 2),\ \ (2, 5),\ \ (4, 1),\ \ (4, 2)$$
$$(4, 5),\ \ (4, 4)\ \ (4, 3)$$

and the tableau at the completion of this stage is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 / 0	8 / 0	9 / 0	4 • / 9	6 / 0	9
2	5 / 0	10 / 0	8 / 0	9 / 0	5 • / 14	14
3	6 / 0	4 • / 5	12 / 0	12 / 0	4 / 0	5
4	5 • / 1	7 • / 3	12 • / 6	10 • / 5	8 • / 1	16
5	4 • / 7	6 / 0	8 / 0	10 / 0	12 / 0	7
6	7 / 0	3 • / 9	7 / 0	12 / 0	8 / 0	9
d_j	8	17	6	14	15	60

The Transportation Problem: The Minimum Cost Method (continued)

We have now found an initial basic feasible solution to this transportation problem. This initial basic feasible solution is determined by basis B, where

$$\begin{aligned} B &= \{(6,2),\ (1,4),\ (5,1),\ (3,2),\ (2,5),\ (4,1), \\ &\qquad (4,2),\ (4,5),\ (4,4)\ (4,3)\}. \end{aligned}$$

Note that $x_{i,j} = 0$ when $(i,j) \notin B$. This corresponds to the requirement that $(x_{i,j})$ be a basic feasible solution determined by the basis B. Also $x_{i,j} > 0$ for all $(i,j) \in B$.

The Transportation Problem: The Minimum Cost Method (continued)

We have thus found an initial basic feasible solution, given by $x_{i,j} = (X)_{i,j}$, where X is the following 6×5 matrix:—

$$X = \begin{pmatrix} 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 14 \\ 0 & 5 & 0 & 0 & 0 \\ 1 & 3 & 6 & 5 & 1 \\ 7 & 0 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 \end{pmatrix}.$$

The cost of this initial feasible basic solution is

$$\begin{aligned} & 4 \times 9 + 5 \times 14 + 4 \times 5 + 5 \times 1 + 7 \times 3 \\ & \qquad + 12 \times 6 + 10 \times 5 + 8 \times 1 + 4 \times 7 + 3 \times 9 \\ = \; & 36 + 70 + 20 + 5 + 21 + 72 + 50 + 8 + 28 + 27 \\ = \; & 337. \end{aligned}$$

The average transportation cost per unit of the commodity is then 5.617.

We next determine whether the initial basic feasible solution found by the Minimum Cost Method is an optimal solution, and, if not, how to adjust the basis go obtain a solution of lower cost.

We determine $u_1, u_2, u_3, u_4, u_5, u_6$ and v_1, v_2, v_3, v_4, v_5 such that $c_{i,j} = v_j - u_i$ for all $(i, j) \in B$, where B is the initial basis, specified as follows:

$$\begin{aligned} B &= \{(6,2),\ (1,4),\ (5,1),\ (3,2),\ (2,5),\ (4,1),\\ &\qquad (4,2),\ (4,5),\ (4,4)\ (4,3)\}. \end{aligned}$$

We seek a solution with $u_1 = 0$. We then determine $q_{i,j}$ so that $c_{i,j} = v_j - u_i + q_{i,j}$ for all i and j.

We therefore complete the following tableau:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow q_{i,j}$	1	2	3	4	5	u_i
1	12 ?	8 ?	9 ?	4 • 0	6 ?	0
2	5 ?	10 ?	8 ?	9 ?	5 • 0	?
3	6 ?	4 • 0	12 ?	12 ?	4 ?	?
4	5 • 0	7 • 0	12 • 0	10 • 0	8 • 0	?
5	4 • 0	6 ?	8 ?	10 ?	12 ?	?
6	7 ?	3 • 0	7 ?	12 ?	8 ?	?
v_j	?	?	?	?	?	

Now $(1,4) \in B$, $u_1 = 0$ and $c_{1,4} = 4$ force $v_4 = 4$. Then $(4,4) \in B$, $v_4 = 4$ and $c_{4,4} = 10$ force $u_4 = -6$. After entering these values, the tableau is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow q_{i,j}$	1	2	3	4	5	u_i
1	12 ?	8 ?	9 ?	4 • 0	6 ?	0
2	5 ?	10 ?	8 ?	9 ?	5 • 0	?
3	6 ?	4 • 0	12 ?	12 ?	4 ?	?
4	5 • 0	7 • 0	12 • 0	10 • 0	8 • 0	-6
5	4 • 0	6 ?	8 ?	10 ?	12 ?	?
6	7 ?	3 • 0	7 ?	12 ?	8 ?	?
v_j	?	?	?	4	?	

Next we note that $(4, j) \in B$ for all j. Therefore $u_4 = -6$ and $c_{4,j} = v_j - u_4$ force $v_j = c_{4,j} - 6$ for all j. Therefore $v_1 = -1$, $v_2 = 1$, $v_3 = 6$ and $v_5 = 2$. After entering these values, the tableau is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow q_{i,j}$	1	2	3	4	5	u_i
1	12 ?	8 ?	9 ?	4 • 0	6 ?	0
2	5 ?	10 ?	8 ?	9 ?	5 • 0	?
3	6 ?	4 • 0	12 ?	12 ?	4 ?	?
4	5 • 0	7 • 0	12 • 0	10 • 0	8 • 0	-6
5	4 • 0	6 ?	8 ?	10 ?	12 ?	?
6	7 ?	3 • 0	7 ?	12 ?	8 ?	?
v_j	-1	1	6	4	2	

Next $(2,5) \in B$, $c_{2,5} = 5$ and $v_5 = 2$ forces $u_2 = -3$.

Also $(3,2) \in B$, $c_{3,2} = 4$ and $v_2 = 1$ forces $u_3 = -3$.

Also $(5,1) \in B$, $c_{5,1} = 4$ and $v_1 = -1$ forces $u_5 = -5$.

Also $(6,2) \in B$, $c_{6,2} = 3$ and $v_2 = 1$ forces $u_6 = -2$.

After entering these values, the tableau is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow q_{i,j}$	1	2	3	4	5	u_i
1	12 ?	8 ?	9 ?	4 • 0	6 ?	0
2	5 ?	10 ?	8 ?	9 ?	5 • 0	-3
3	6 ?	4 • 0	12 ?	12 ?	4 ?	-3
4	5 • 0	7 • 0	12 • 0	10 • 0	8 • 0	-6
5	4 • 0	6 ?	8 ?	10 ?	12 ?	-5
6	7 ?	3 • 0	7 ?	12 ?	8 ?	-2
v_j	-1	1	6	4	2	

We have thus found the following values for the u_i and v_j:

$$u_1 = 0, \;\; u_2 = -3, \;\; u_3 = -3, \;\; u_4 = -6, \;\; u_5 = -5, \;\; u_6 = -2.$$

$$v_1 = -1, \;\; , v_1 = 1, \;\; v_2 = 6, \;\; v_4 = 4, \;\; v_5 = 2.$$

We next calculate $q_{i,j}$ for all i and j so that $c_{i,j} = v_j - u_i + q_{i,j}$. Note that the determination of the numbers u_i and v_j ensures that $q_{i,j} = 0$ for all $(i, j) \in B$.

After entering the value of $q_{i,j}$ for all i and j, the tableau is as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow q_{i,j}$	1	2	3	4	5	u_i
1	12	8	9	4 •	6	0
	13	7	3	0	4	
2	5	10	8	9	5 •	-3
	3	6	-1	2	0	
3	6	4 •	12	12	4	-3
	4	0	3	5	-1	
4	5 •	7 •	12 •	10 •	8 •	-6
	0	0	0	0	0	
5	4 •	6	8	10	12	-5
	0	0	-3	1	5	
6	7	3 •	7	12	8	-2
	6	0	-1	6	4	
v_j	-1	1	6	4	2	

The initial basic feasible solution is not optimal because some of the quantities $q_{i,j}$ are negative. Indeed $q_{2,3} = -1$, $q_{3,5} = -1$, $q_{5,3} = -3$ and $q_{6,3} = -1$. The most negative of these is $q_{5,3}$. We therefore seek to bring $(5, 3)$ into the basis.

The procedure for achieving this requires us to determine a 6×5 matrix Y satisfying the following conditions:—

- $y_{5,3} = 1$;
- $y_{i,j} = 0$ when $(i, j) \notin B \cup \{(5, 3)\}$;
- all rows and columns of the matrix Y sum to zero.

Accordingly we fill in the following tableau with those coefficients $y_{i,j}$ of the matrix Y that correspond to cells in the current basis (marked with the • symbol), so that all rows sum to zero and all columns sum to zero:—

$y_{i,j}$	1	2	3	4	5	
1				? •		0
2					? •	0
3		? •				0
4	? •	? •	? •	? •	? •	0
5	? •		1 ∘			0
6		? •				0
	0	0	0	0	0	0

The constraint that the rows and columns of the table all sum to zero determines $y_{4,3}$, and also determines $y_{i,j}$ for all $(i,j) \in B$ satisfying $i \neq 4$. These values of $y_{i,j}$ are recorded in the following tableau:—

$y_{i,j}$	1		2		3		4		5		
1							0	•			0
2									0	•	0
3			0	•							0
4	?	•	?	•	-1	•	?	•	?	•	0
5	-1	•			1	◦					0
6			0	•							0
	0		0		0		0		0		0

The remaining values of $y_{i,j}$ for $(i,j) \in B$ are then readily determined, and the tableau is completed as follows:—

The Transportation Problem: The Minimum Cost Method (continued)

$y_{i,j}$	1		2		3		4		5		
1							0	•			0
2									0	•	0
3			0	•							0
4	1	•	0	•	-1	•	0	•	0	•	0
5	-1	•			1	◦					0
6			0	•							0
	0		0		0		0		0		0

We now determine those values of λ for which $X + \lambda Y$ is a feasible solution, where

$$X + \lambda Y = \begin{pmatrix} 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 14 \\ 0 & 5 & 0 & 0 & 0 \\ 1+\lambda & 3 & 6-\lambda & 5 & 1 \\ 7-\lambda & 0 & \lambda & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 \end{pmatrix}.$$

From this matrix, it is clear that $X + \lambda Y$ is a feasible solution for $0 \leq \lambda \leq 6$. Moreover the next basis is obtained by adding $(5, 3)$ to the existing basis and removing $(4, 3)$. The new basic feasible solution corresponding to the new basis is obtained from $X + \lambda Y$ by setting $\lambda = 6$.

We now let B denote the new basis and let X denote the new basic feasible solution corresponding to the new basis. Accordingly

$$\begin{aligned} B \;=\; & \{(6,2),\;(1,4),\;(5,1),\;(3,2),\;(2,5),\;(4,1), \\ & (4,2),\;(4,5),\;(4,4)\;(5,3)\}. \end{aligned}$$

and

$$X = \begin{pmatrix} 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 14 \\ 0 & 5 & 0 & 0 & 0 \\ 7 & 3 & 0 & 5 & 1 \\ 1 & 0 & 6 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 \end{pmatrix}.$$

Moreover

$$\text{Cost} = \text{Old Cost} + 6 * (-3) = 337 - 18 = 319.$$

The cost of the current feasible solution can also be obtained from the data recorded in the following tableau that represents the current feasible solution:—

The Transportation Problem: The Minimum Cost Method (continued)

$c_{i,j} \searrow x_{i,j}$	1	2	3	4	5	s_i
1	12 / 0	8 / 0	9 / 0	4 • / 9	6 / 0	9
2	5 / 0	10 / 0	8 / 0	9 / 0	5 • / 14	14
3	6 / 0	4 • / 5	12 / 0	12 / 0	4 / 0	5
4	5 • / 7	7 • / 3	12 / 0	10 • / 5	8 • / 1	16
5	4 • / 1	6 / 0	8 • / 6	10 / 0	12 / 0	7
6	7 / 0	3 • / 9	7 / 0	12 / 0	8 / 0	9
d_j	8	17	6	14	15	60