

Module MA3428, Hilary Term 2015

Preparing for Examination

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1. Rings and Modules

You will not be examined directly on the definitions, statements of results and proofs in this section. All the material in this section should correspond to material covered in abstract algebra courses in the Freshman years, though maybe stated in the context of Abelian groups (special cases of results for groups) or vector spaces. Abelian groups and vector spaces are standard examples of modules, and generalizing to left modules adds nothing essentially new.

The results of this section are used regularly throughout the module.

2. Free Modules

You will not be examined directly on the definitions, statements of results and proofs in this section. You need however to have an understanding of what is meant by saying that a left module M over a unital ring R is *free*, the concepts of linear dependence and independence in the context of left modules over a unital ring, and what is meant by saying that a subset of a module is a *free basis* of that module (and thus freely generates the module). You should also understand the result stated in Proposition 2.4 (the “universal property” that characterizes free left modules over a unital ring up to isomorphism) so as to be able to apply this result in the contexts where the result is used in subsequent sections of the module.

3. Simplicial Complexes

This material is examinable. In consulting past papers, you should note however that the amount of material is reduced considerably compared to what

was taught in previous years, and therefore candidates attempting the examination in earlier years would have seen results and examples not included in the module this year.

4. The Chain Groups of a Simplicial Complex

You need to understand the statements of Lemma 4.1, Lemma 4.2, Proposition 4.4 and Proposition 4.5, and be able to apply them in the context in which these results are applied in subsequent sections of the module. The proofs of these results, and of other results in this section, are non-examinable. Also the statement and proof of Lemma 4.3 is non-examinable.

5. The Homology Groups of a Simplicial Complex

Regarding subsection 5.1, you need to understand in particular that there are two possible orientations on a simplex of positive dimension, that an orientation on a simplex of positive dimension can be specified by listing the vertices of that simplex in some order, and that orientation is preserved under even permutations of the ordering and is reversed under odd permutations of the ordering. However the statement and proof of Proposition 5.1 and the details of the following example are non-examinable.

Subsections 5.2 and 5.3 are examinable.

6. Homology Calculations

You should be able to derive information about homology groups from specifications of simplicial complexes (where the number of simplices is not excessively large), as in the examples of this section. (The two examples in this section are in fact derived from old examination questions.) Many examination questions similar to the examples in this section can be found in past examination papers for Modules MA3428 and Module MA421 (the predecessor module for MA3427 and MA3428). Worked solutions covering many of these papers can be accessed from the webpages for MA3428 and MA421.

7. The Homology Groups of Filled Polygons

This section is new to MA3428. You should be familiar with the results concerning the homology of simple polygonal chains and simple polygons presented in subsections 7.1 and 7.2. The statements and proofs of results concerning two-dimensional simplicial complexes in subsections 7.3, 7.4 and

7.5 are non-examinable (though it would be permissible to cite these results in examples in appropriate contexts).

8. General Theorems concerning the Homology of Simplicial Complexes

The results of this section are examinable, with the exception of subsection 8.5 on reduced homology groups.

9. Introduction to Homological Algebra

The material in this section is examinable.

10. The Mayer-Vietoris Exact Sequence

You should be familiar with the Mayer-Vietoris Exact Sequence, and be able to apply it to obtain information about the homology groups of simplicial complexes.

With regard to the examples regarding the torus, the details of triangulations (with labellings of vertices, the embedding of the torus in three-dimensional space) etc. are non-examinable. In studying these examples it is advisable to focus on the algebraic aspects that explain how information on the homology groups of appropriate subcomplexes and the induced homomorphisms between those subcomplexes is used in connection with the Mayer-Vietoris sequence to obtain information about the relevant simplicial complex.

Problems based on applications of the Mayer-Vietoris sequence appeared regularly on examination papers in Module MA421 up to 2005. The Mayer-Vietoris sequence was presented in the lecture course but was not examined in 2007. From 2009 onwards, the use of exact sequences for obtaining information about homology groups was extensively discussed in lectures and subsequently examined based on the use of another important exact sequence: the exact sequence of homology groups associated with a simplicial pair (which was not discussed in 2015, but which was a significant part of the content of the relevant module in 2009, 2011, and 2013).