Notes regarding Examinable Material from MA2C03 Hilary Term at the Annual Examination 2016

David R. Wilkins

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Spannng Trees (Sections 32 to 35)

Candidates should in particular be able to give proofs of results in these sections concerning trees, and concerning spanning trees in connected graphs, and should be able to apply both Kruskal's algorithm and Prim's algorithm to find minimal spanning trees of given connected graphs (with costs assigned to the edges of those graphs).

The proofs of the following results in these sections are non-examinable:—

- Proposition 34.1;
- Theorem 34.2;
- Proposition 35.1;
- Theorem 35.2.

Vectors (Sections 36 to 38)

These sections include results on vectors in three-dimensional space. Candidates should be able in particular to calculate the length of a vector, the cosine of the angle between two vectors, the equation of a plane passing through any three given non-collinear points, and the volumes of parallelepipeds.

Proofs of the following results in this section are non-examinable:—

- Lemma 36.1
- Theorem 36.2

- Proposition 38.1
- Proposition 38.3
- Proposition 38.5

Non-Examinable Propositions

The proofs of the following results are non-examinable:

Ordinary Differential Equations (Section 39)

Candidates should be able to prove the followiing results concerning the ordinary differential equation

$$\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0 :$$

- if the auxiliary polynomial $z^2 + bz + c$ has two distinct real roots r and s then $Ae^{rx} + Be^{sx}$ is a solution of the differential equation for all real constants A and B;
- if the auxiliary polynomial $z^2 + bz + c$ has a repeated real root r then $(A+Bx)e^{rx}$ is a solution of the differential equation for all real constants A and B;
- if the auxiliary polynomial $z^2 + bz + c$ has non-real roots $p + \sqrt{-1}\, q$ and $p - \sqrt{-1}\, q$, where p and q are real numbers, then $e^{px}(A\cos qx + B\sin qx)$ is a solution of the differential equation for all real constants A and B.

(Candidates are not required to prove that these are the *only* solutions of the differential equations in the cases specified above.)

Candidates should also be able to solve the inhomogeneous second order linear differential equations wiith constant coefficients considered in subsection 39.2. This involves deriving the complete solutions to differential equations of the following forms, given appropriate information on the roots of the auxiliary polynomial $z^2 + bz + c$, where a, b, c, g, h, k and m are real constants:—

$$\begin{aligned}
\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy &= g + hx + kx^2 \\
\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy &= (g + hx)e^{mx} \\
\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy &= g\cos kx + h\sin kx.
\end{aligned}$$

Proofs of the following results in this section are non-examinable:—

- Proposition 39.1
- Proposition 39.2
- Proposition 39.3
- Proposition 39.4
- Proposition 39.5
- Theorem 39.6
- Proposition 39.7

Introduction to Harmonic Analysis (Section 40)

This section is not examinable.

Introduction to Number Theory and Cryptoography (Sections 41 and 42)

Candidates should in particular be able to apply the Chinese Remainder Theorem (Theorem 41.16) in cases where the moduli are less than 100.

Also, given a positive integer a and a reasonably small prime number p less than 100, and given an integer n, candidates should be able to determine the unique integer x satisfying $0 \leq x < p$ for which $a^n \equiv x \pmod{p}$.

The proofs of results in subsections 41.1 to 41.7 will not be examinable in themselves, but candidates should be familiar with the definitions and basic results concerning prime numbers and congruences, as these are used in material and problems that are examinable.

In addition, proofs of the following results are not examinable:—

- Theorem 41.17
- Lemma 41.18
- Theorem 42.1