Course MA2C03: Michaelmas Term 2014.

Assignment I.

To be handed in by Friday 21st November, 2014.
Please include both name and student number on any work handed in.

1. Let A, B and C be sets. Prove that

$$(A \setminus B) \cap C = A \cap (C \setminus B).$$

2. Let S be the relation on the set $\mathbb{Z}$ of integers, where integers x and y satisfy xSy if and only if $x^2 + y^2$ is divisible by 2. Determine

   (i) whether or not the relation S is *reflexive*,

   (ii) whether or not the relation S is *symmetric*,

   (iii) whether or not the relation S is *anti-symmetric*,

   (iv) whether or not the relation S is *transitive*,

   (v) whether or not the relation S is a *equivalence relation*,

   (vi) whether or not the relation S is a *partial order*.

   [Justify your answers with short proofs and/or counterexamples.]

3. Let $f\colon [0,3] \to [0,9]$ be the function defined so that $f(x) = x^3 - 3x^2 + 3x$ for all $x \in [0,3]$. Determine whether or not this function is injective, and whether or not it is surjective, giving brief reasons for your answers. (Note that $[0,3]$ denotes the set of all *real numbers* between 0 and 3 inclusive.)

4. Let $\mathbb{R}$ be the set of all real numbers, and let $*$ be the binary operation on $\mathbb{R}$ defined such that

$$x * y = 3xy + 2x + 2y + \frac{2}{3}.$$

   for all $x, y \in \mathbb{R}$. Prove that $(\mathbb{R}, *)$ is a monoid. What is the identity element of this monoid? Determine which elements of the monoid are invertible. Is the monoid $(\mathbb{R}, *)$ a group?