Course MA2C03: Michaelmas Term 2013.

Assignment II.

To be handed in by Wednesday 22nd January, 2014.
Please include both name and student number on any work handed in.

1. Let $\mathbb{R}^2$ be the set of all ordered pairs of numbers, and let $\otimes$ be the binary operation on $\mathbb{R}^2$ defined such that

$$(a, b) \otimes (c, d) = (a + c, ac + b + d)$$

for all $(a, b), (c, d) \in \mathbb{R}^2$. Prove that $(\mathbb{R}^2, \otimes)$ is a monoid. What is the identity element of this monoid? Is the monoid $(\mathbb{R}^2, \otimes)$ a group?

2. (a) Describe the formal language over the alphabet $\{0, 1\}$ generated by the context-free grammar whose only non-terminal is $\langle S\rangle$, whose start symbol is $\langle S\rangle$ and whose productions are the following:

$$\begin{array}{lcl} \langle S\rangle & \to & 1 \\ \langle S\rangle & \to & 0\langle S\rangle \end{array}$$

(i.e., describe the structure of the binary strings generated by the grammar). Is this context-free grammar a regular grammar?

(b) Give the specification of a finite state acceptor that accepts the language over the alphabet $\{a, b, c\}$ consisting of all words, including the empty word, where the string aba never occurs as a substring. (In particular you should specify the set of states, the starting state, the finishing states, and the transition table that determines the transition function of the finite state acceptor.)

(c) Devise a regular grammar to generate the language specified in (b). (In particular, you should specify the nonterminals, the start symbol and the productions of the grammar.)

3. (a) Let (V, E) be the graph with vertices a, b, c, d, e and edges $a\,b$, $a\,e$, $b\,c$, $b\,e$, $c\,d$, $d\,e$.

   (i) Is the graph complete?

   (ii) Is the graph connected?

(iii) Is the graph regular?

(iv) Does the graph have an Eulerian trail?

(v) Does the graph have an Eulerian circuit?

(vi) Does the graph have a Hamiltonian circuit?

(vii) Is the graph a tree?

(b) Give an example of an isomorphism from the graph (V, E) specified in (a) to the graph (V_1, E_1) with vertices p, q, r, s, t and edges and edges $p\,q$, $p\,s$, $q\,t$, $r\,s$, $r\,t$, $s\,t$.