Course MA2C03: Michaelmas Term 2013.

Assignment I.

To be handed in by Tuesday 29th October, 2013.
Please include both name and student number on any work handed in.

1. Use the Principle of Mathematical Induction to prove that

$$\sum_{k=1}^{n} 7^k k = \frac{7}{36}\Big((6n-1)7^n + 1\Big)$$

   for all positive integers n.

2. Let A and B be sets. Prove that

$$(A \cup B) \setminus (A \setminus B) = B.$$

3. Let S be the relation on the set $\mathbb{Z}$ of integers, where integers x and y satisfy xSy if and only if $x^3 - x \geq y^3 - y$. Determine

   (i) whether or not the relation S is *reflexive*,

   (ii) whether or not the relation S is *symmetric*,

   (iii) whether or not the relation S is *anti-symmetric*,

   (iv) whether or not the relation S is *transitive*,

   (v) whether or not the relation S is a *equivalence relation*,

   (vi) whether or not the relation S is a *partial order*.

   [Justify your answers with short proofs and/or counterexamples.]

4. Let $f\colon [1,4] \to [0,6]$ be the function defined so that $f(x) = x^2 - 4x + 4$ for all $x \in [1,4]$. Determine whether or not this function is injective, and whether or not it is surjective, giving brief reasons for your answers. (Note that $[1,4]$ denotes the set of all *real numbers* between 1 and 4 inclusive, and therefore includes fractions such as $\frac{3}{2}$ and irrational numbers like $\sqrt{2}$ and π.)