Course MA2C02: Hilary Term 2011.

Assignment IV.

To be handed in by Friday 25th March, 2011.
Please include both name and student number on any work handed in.

1. Let $f\colon [0,1] \to \mathbb{R}$ be the real-valued function on the interval $[0,1]$ defined as follows:
$$f(x) = \begin{cases} 4x & \text{if } 0 \le x \le \frac{1}{4}: \\ \frac{4}{3}(1-x) & \text{if } \frac{1}{4} \le x \le 1. \end{cases}$$
This function may be represented as a Fourier series of the form
$$f(x) = \sum_{n=1}^{+\infty} b_n \sin n\pi x,$$
where
$$\begin{aligned} b_n &= 2\int_0^1 f(x) \sin n\pi x \, dx \\ &= 2\int_0^{\frac{1}{4}} f(x) \sin n\pi x \, dx + 2\int_{\frac{1}{4}}^1 f(x) \sin n\pi x \, dx. \end{aligned}$$
Find the values of the coefficients b_n for $n = 1, 2, 3, 4, \ldots$. (Note that
$$\sin\frac{1}{4}\pi = \cos\frac{1}{4}\pi = \frac{1}{\sqrt{2}},$$
$\sin\theta = \cos(\frac{1}{2}\pi - \theta)$ for all real numbers θ, $\sin\frac{1}{2}\pi = -\sin\frac{3}{2}\pi = 1$, and
$$\sin\frac{1}{4}\pi = \sin\frac{3}{4}\pi = -\sin\frac{5}{4}\pi = -\sin\frac{7}{4}\pi, \text{ etc.})$$

2. Find the lengths of the vectors $(3,6,6)$ and $(4,4,7)$ and also the cosine of the angle between them.

3. Calculate the components of a non-zero vector (a,b,c) in $\mathbb{R}^3$ that is orthogonal to the vectors $(1,1,2)$ and $(2,3,2)$.

4. Let the quaternions q and r be defined as follows:
$$q = 1 - i - 2k, \quad r = j - 3k.$$
Calculate the quaternion products qr and rq. [Hamilton's basic formulae for quaternion multiplication state that
$$i^2 = j^2 = k^2 = -1, \quad ij = -ji = k, \quad jk = -kj = i, \quad ki = -ik = j.]$$