Course MA2C02: Hilary Term 2011.

Assignment III.

To be handed in by Wednesday 9th March, 2011.
Please include both name and student number on any work handed in.

1. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = \cos 3x.$$

2. Any function y of a real variable x that solves the differential equation

$$\frac{d^4y}{dx^4} - 16y = 0$$

may be represented by a power series of the form

$$y = \sum_{n=0}^{+\infty} \frac{y_n}{n!} x^n,$$

where the coefficients $y_0, y_1, y_2, y_3, \ldots$ of this power series are real numbers.

Find values of these coefficients y_n for $n = 0, 1, 2, 3, 4, \ldots$ that yield a solution to the above differential equation with $y_0 = 1$ and $y_1 = 0$ and $y_2 = -4$ and $y_3 = 0$. Hence or otherwise, find the solution to this differential equation.

3. Let $(z_n : n \in \mathbb{Z})$ be the doubly-infinite 3-periodic sequence with $z_0 = 1$, $z_1 = 2$, $z_2 = -2$. Find values of c_0, c_1, c_2 such that

$$z_n = c_0 + c_1\omega^n + c_2\omega^{2n}$$

for all integers n, where $i = \sqrt{-1}$ and

$$\omega = e^{2\pi i/3} = \cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3} = \frac{1}{2}(-1 + \sqrt{3}\, i).$$