## Course MA2C02: Hilary Term 2010. Assignment III.

## To be handed in by Wednesday 10th March, 2010. Please include both name and student number on any work handed in.

1. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = x^2 e^{2x}.$$

2. Let  $f: \mathbb{R} \to \mathbb{R}$  be the function with period 6 whose values on the interval [0, 6] are defined as follows:

$$f(x) = \begin{cases} 2x & \text{if } 0 \le x \le 3; \\ 12 - 2x & \text{if } 3 \le x \le 6. \end{cases}$$

Express the function f as a Fourier series of the form

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{\pi nx}{3}.$$

3. Let  $(z_n : n \in \mathbb{Z})$  be the doubly-infinite 4-periodic sequence with  $z_0 = 1$ ,  $z_1 = 2$ ,  $z_2 = 3i$  and  $z_3 = -1 - i$ . Find values of  $c_0$ ,  $c_1$ ,  $c_2$  and  $c_3$  such that

$$z_n = c_0 + c_1 i^n + c_2 (-1)^n + c^3 (-i)^n$$

for all integers n.