Course MA2C01: Michaelmas Term 2012.

Assignment II.

To be handed in by Wednesday 23st January, 2013.
Please include both name and student number on any work handed in.

1. Let $\mathbb{R}^3$ be the set of all ordered triples of numbers, and let $\otimes$ be the binary operation on $\mathbb{R}^3$ defined such that

$$(x_1, y_1, z_1) \otimes (x_2, y_2, z_2) = (x_1x_2, x_1y_2 + y_1z_2, z_1z_2)$$

for all $(x_1, y_1, z_1), (x_2, y_2, z_2) \in \mathbb{R}^3$. Prove that $(\mathbb{R}^3, \otimes)$ is a monoid. What is the identity element of this monoid? Is the monoid $(\mathbb{R}^3, \otimes)$ a group?

2. (a) Describe the formal language over the alphabet $\{0, 1\}$ generated by the context-free grammar whose only non-terminal is $\langle S \rangle$, whose start symbol is $\langle S \rangle$ and whose productions are the following:

$$\begin{array}{rcl} \langle S \rangle & \to & 0 \\ \langle S \rangle & \to & 1\langle S \rangle 1 \end{array}$$

(i.e., describe the structure of the binary strings generated by the grammar). Is this context-free grammar a regular grammar?

(b) Give the specification of a finite state acceptor that accepts the language over the alphabet $\{0, 1\}$ consisting of all words where the number of occurrences of the digit 0 within the word is a multiple of 3. (In particular you should specify the set of states, the starting state, the finishing states, and the transition table that determines the transition function of the finite state acceptor.)

(c) Devise a regular grammar to generate the language specified in (b). (In particular, you should specify the nonterminals, the start state and the productions of the grammar.)

3. (a) For each of the following graphs, answer the following questions (giving brief justifications for your answers):

   - Is the graph connected?

- Is the graph regular?
- Does the graph have an Eulerian trail?
- Does the graph have an Eulerian circuit?
- Does the graph have a Hamiltonian circuit?
- Is the graph a tree?

(i) The graph (V, E_1), where $V = \{a, b, c, d, e\}$ and

$$E_1 = \{a\,e,\ b\,c,\ b\,d,\ c\,d\};$$

(ii) The graph (V, E_2), where $V = \{a, b, c, d, e\}$ and

$$E_2 = \{a\,e,\ b\,c,\ b\,d,\ b\,e\};$$

(iii) The graph (V, E_3), where $V = \{a, b, c, d, e\}$ and

$$E_3 = \{a\,b,\ a\,d,\ b\,c,\ c\,d,\ d\,e\}.$$

(iv) The graph (V, E_4), where $V = \{a, b, c, d, e\}$ and

$$E_4 = \{a\,b,\ a\,d,\ b\,c,\ b\,d,\ b\,e,\ c\,d,\ d\,e\}.$$

(b) Give an example of an isomorphism from the graph (V, E_4) specified in (iv) above to the graph (V', E') where $V' = \{p, q, r, s, t\}$ and

$$E' = \{p\,q,\ p\,r,\ p\,s,\ p\,t,\ q\,r,\ q\,s,\ q\,t\}.$$