Course MA2C01: Michaelmas Term 2011.

Assignment II.

To be handed in by Wednesday 1st February, 2012.
Please include both name and student number on any work handed in.

1. (a) Let A be the set $\mathbb{C} \times \mathbb{C}$ consisting of all ordered pairs (z, w), where z and w are complex numbers. Let $\times$ denote the binary operation on A defined by $(z, w) \times (u, v) = (zu - wv, zv + wu)$ for all complex numbers z, w, u and v. Prove that $(A, \times)$ is a monoid. What is its identity element? Prove that an element (z, w) of A is invertible if and only if $z^2 + w^2 \neq 0$.

   (b) Let $(\mathbb{C}, \times)$ be the monoid consisting of the set of complex numbers with the usual operation of multiplication, and let $f\colon \mathbb{C} \to A$ be the function from $\mathbb{C}$ to A which sends the complex number $x + iy$ to the ordered pair (x, y) for all real numbers x and y. Is the function f a homomorphism from $(\mathbb{C}, \times)$ to $(A, \times)$? Is this function an isomorphism?

2. (a) Describe the formal language over the alphabet $\{0, 1\}$ generated by the context-free grammar whose only non-terminal is $\langle S\rangle$, whose start symbol is $\langle S\rangle$ and whose productions are the following:

$$\begin{aligned} \langle S\rangle &\to 0 \\ \langle S\rangle &\to 00\langle S\rangle \\ \langle S\rangle &\to \langle S\rangle 11 \end{aligned}$$

   Is this context-free grammar a regular grammar?

   (b) Give the specification of a finite state acceptor for the language over the alphabet $\{a, b, c\}$ consisting of all finite strings, such as *aabbc*, *aabbbc* and *aaabbbc*, that consist of two or more occurrences of the character a, followed by two or more occurrences of the character b, followed by a single occurrence of the character c. You should in particular specify the starting state, the finishing state or states, and the transition table for this finite state acceptor.

   (c) Give the specification of a regular grammar to generate the language over the alphabet $\{a, b, c\}$ that was defined in (b).

3. Consider a graph G with vertices a, b, c, d and e, and edges $a\,b$, $a\,d$, $b\,c$, $b\,d$, $c\,d$, $c\,e$ and $d\,e$. Let V denote the set of vertices of this graph, so that $V = \{a, b, c, d, e\}$.

(a) Draw a diagram showing the vertices and edges of this graph.

(a) Determine the degrees of each of the vertices of the graph.

(c) Is this graph regular? [If not, briefly explain why not.]

(d) Is this graph complete? [If not, briefly explain why not.]

(e) Is this graph connected? [If not, briefly explain why not.]

(f) Does this graph have an Eulerian circuit? [If so, give an example. If not, briefly explain why not.]

(g) Does this graph have an Eulerian trail that starts at some vertex of the graph, and ends at some other vertex? [If so, give an example. If not, briefly explain why not.]

(h) Does this graph have a Hamiltonian circuit? [If so, give an example.]

(i) Is this graph a tree? [If not, briefly explain why not.]

(j) Does this graph have a spanning tree? [If so, give an example. If not, briefly explain why not.]

(k) Does there exist a function $\theta\colon V \to V$ from the set V of vertices of the graph to itself that is an isomorphism from the graph G to itself and that satisfies $\theta(a) = e$. [If so, give an example. If not, briefly explain why not.]

(l) Does there exist a function $\varphi\colon V \to V$ from the set V of vertices of the graph to itself that is an isomorphism from the graph G to itself and that satisfies $\varphi(c) = d$. [If so, give an example. If not, briefly explain why not.]