# Course MA2C01: Michaelmas Term 2009.

# Assignment II.

**To be handed in by Wednesday 3rd February, 2010.**
**Please include both name and student number on any work handed in.**

1. Let $c$ be a fixed positive integer, and let $\otimes$ denote the binary operation on the set $\mathbb{Z}$ of integers defined by the formula

$$x \otimes y = xy + c(x + y) + c^2 - c$$

   for all integers $x$, $y$ and $z$.

   (a) Is $(\mathbb{Z}, \otimes)$ a semigroup? [Justify your answer.]

   (b) Is $(\mathbb{Z}, \otimes)$ a monoid? If so, what is its identity element?

   (c) Which of the elements of $\mathbb{Z}$ are invertible? Is $(\mathbb{Z}, \otimes)$ a group?

2. Construct a regular grammar that generates the language $L$ over the alphabet $\{0, 1\}$, where

$$L = \{1, 1000, 1000000, 1000000000, \ldots\},$$

   so that a string of binary digits belongs to $L$ if and only if it consists of the digit 1 followed by a string of $3n$ zeroes, for some non-negative integer $n$. You should specify your formal grammar in Backus-Naur form.

3. Answer the following questions concerning the graph with vertices $a$, $b$, $c$, $d$, $e$ and $f$ pictured above. [Justify all your answers.]

   (a) Is the graph complete?

   (b) Is the graph regular?

   (c) Is the graph connected?

   (d) Does the graph have an Eulerian circuit?

   (e) Does the graph have a Hamiltonian circuit?

   (f) Give an example of a spanning tree for the graph, specifying the vertices and edges of the spanning tree.

   (g) Given an example of an isomorphism between the graph pictured above and that pictured below. (You should specify the isomorphism as a function between the sets $\{a, b, c, d, e, f\}$ and $\{u, v, w, x, y, z\}$ of vertices of the two graphs.)