Course MA2C01: Michaelmas Term 2012.

Assignment I.

To be handed in by Wednesday 31st October, 2012.
Please include both name and student number on any work handed in.

1. Use the Principle of Mathematical Induction to prove that

$$\sum_{i=0}^{n} x^i = \frac{1 - x^{n+1}}{1 - x}$$

for all positive integers n and real numbers x satisfying $x \neq 1$.

2. Let A, B and C be sets. Prove that

$$(A \setminus C) \cup (B \setminus C) = (A \cup B) \setminus C.$$

3. Let Q be the relation on the set $\mathbb{R}*$ of non-zero real numbers, where non-zero real numbers x and y satisfy xQy if and only if $\dfrac{x^2}{y^2}$ is an rational number. Determine

   (i) whether or not the relation Q is *reflexive*,

   (ii) whether or not the relation Q is *symmetric*,

   (iii) whether or not the relation Q is *anti-symmetric*,

   (iv) whether or not the relation Q is *transitive*,

   (v) whether or not the relation Q is a *equivalence relation*,

   (vi) whether or not the relation Q is a *partial order*.

   Note that a rational number is a number that can be expressed as a fraction n/d whose numerator n and denominator d are integers. Not all real numbers are rational numbers: both $\sqrt{2}$ and π are examples of real numbers that are not rational numbers.

   [Justify your answers with short proofs and/or counterexamples.]

4. Let $f\colon [0, 4] \to [0, 10]$ be the function defined so that

$$f(x) = \begin{cases} x^3 & \text{if } 0 \leq x \leq 2; \\ x + 6 & \text{if } 2 < x \leq 4. \end{cases}$$

Determine whether or not this function is injective, and whether or not it is surjective, giving brief reasons for your answers.