Course MA2C01: Michaelmas Term 2011.

Assignment I.

To be handed in by Wednesday 2nd November, 2011.
Please include both name and student number on any work handed in.

1. Use the Principle of Mathematical Induction to prove that

$$\sum_{i=1}^{n} \frac{1}{i^3} \leq \frac{3}{2} - \frac{1}{2n^2}$$

for all positive integers n.

2. Let A, B and C be sets. Prove that

$$(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B).$$

3. Let Q be the relation on the set $\mathbb{R}$ of real numbers, where real numbers x and y satisfy xQy if and only if e^{x-y} is an integer. Determine

   (i) whether or not the relation Q is *reflexive*,

   (ii) whether or not the relation Q is *symmetric*,

   (iii) whether or not the relation Q is *anti-symmetric*,

   (iv) whether or not the relation Q is *transitive*,

   (v) whether or not the relation Q is a *equivalence relation*,

   (vi) whether or not the relation Q is a *partial order*.

   [Justify your answers with short proofs and/or counterexamples.]

4. Let $f\colon [0,3] \to [0,4]$ be the function defined so that

$$f(x) = -x^2 + 2x + 3.$$

Determine whether or not this function is injective, and whether or not it is surjective, giving brief reasons for your answers.