Course MA2C01: Michaelmas Term 2010.

Assignment I.

To be handed in by Wednesday 17th November, 2010.
Please include both name and student number on any work handed in.

1. Use the Principle of Mathematical Induction to prove that

$$\sum_{i=1}^{n} \frac{2i^2 - 1}{i^4} \leq 4 - \frac{2n+1}{n^2}$$

for all positive integers n.

2. Let A, B and C be sets. Prove that

$$A \cap (B \setminus C) = (A \cap B) \setminus C.$$

3. Let $\sim$ be the relation on the set $\mathbb{R}$ of real numbers, where real numbers x and y satisfy $x \sim y$ if and only if $y^3 - x^3$ is an integer. Determine whether or not the relation $\sim$ is an equivalence relation, and whether or not this relation is a partial order. [Give appropriate short proofs and/or counterexamples to justify your answers.]

4. Let $f\colon [0,2] \to [0,2]$ be the function defined so that

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1; \\ 3 - x & \text{if } 1 \leq x \leq 2. \end{cases}$$

Determine whether or not this function is injective, and whether or not it is surjective, giving brief reasons for your answers.