Preliminary Draft of Statements of Selected Propositions from Book I of Euclid's *Elements of Geometry*, presented in a format suitable for inclusion in Examination Questions.

David R. Wilkins

February 19, 2018

This draft is currently *very* provisional, and it *not* to be taken, at this stage, as definitive with regard to the selection or precise wording of actual examination questions

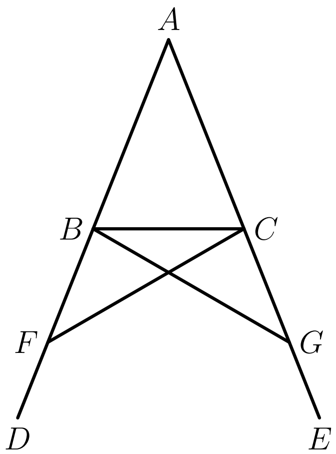
General Note: proofs of results in Euclid should be consistent with the the theory of geometry set out in Euclid's *Elements* (and in particular should be consistent with the ordering of the propositions in each book following the statement of the definitions, postulates and common notions). This means that a proof of a given proposition may cite results from earlier propositions but should not cite results from later propositions. It is not necessary or expected that you reproduce proofs verbatim from any published edition of Euclid, or that you know the precise number of any proposition in the sequence. You may present the proofs freely in your own words.

Based on Euclid, Book I, Propositions 1 and 2

Let A, B and C be points of the (Euclidean) plane that are not collinear. Describe, and justify, a geometrical construction, using only straight-edge and compass, that constructs a point L for which the line segment $[AL]$ is equal to $[BC]$.

Based on Euclid, Book I, Proposition 5

Let $\triangle ABC$ be an isosceles triangle in which the edges $[AB]$ and $[AC]$ are equal, let the edges $[AB]$ and $[AC]$ be produced beyond B and C to D and E respectively. *Without using the result that the angles $\angle ABC$ and $\angle ACB$ are equal to one another, and without using the result of Euclid's* Elements*, Book I, Proposition 13 to the effect that the sum of two supplementary angles is equal to two right angles*, prove that the angles $\angle DBC$ and $\angle ECB$ below the base $[BC]$ of the isosceles triangle are equal to one another.



[It may be useful to make reference to the above figure, in which the line segments $[BF]$ and $[CG]$ are equal to one another. You may use, without proof, the *SAS Congruence Rule*.]

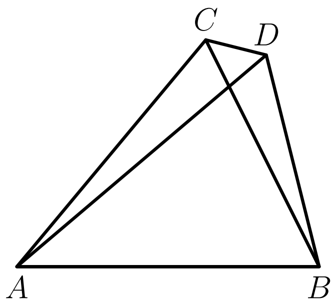
Euclid, Book I, Proposition 6

Prove that if, in a triangle $\triangle ABC$, the angles $\angle ABC$ and $\angle ACB$ are equal to one another, then the sides $[AB]$ and $[AC]$ that subtend the equal angles are equal to one another, and thus the triangle is an isosceles triangle.

[You may use, without proof, one of the conclusions of Euclid's *Elements*, Book I, Proposition 5, which ensures that, in an isosceles triangle, the angles opposite the equal sides are equal to one another.]

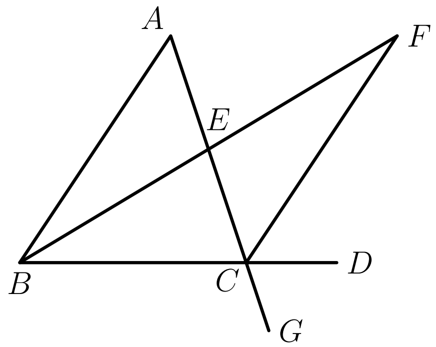
Based on a case of Euclid, Book I, Proposition 7

Let $\triangle ABC$ and $\triangle ABD$ be triangles with a common edge $[AB]$. Suppose that the vertices C and D of those triangles lie on the same side of the line $[AB]$, and that the vertex D lies outside the triangle $\triangle ABC$, as depicted on the figure below. Suppose also that the edges $[AC]$ and $[AD]$ are equal. Prove that the edges $[BC]$ and $[BD]$ are unequal.



Euclid, Book I, Proposition 16

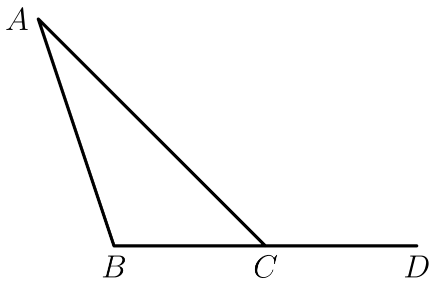
Let a side $[BC]$ of a triangle $\triangle ABC$ be produced beyond C to a point D. Prove that the external angle $\angle ACD$ is greater than the internal and opposite angle $\angle CAB$. Prove also that the external angle $\angle ACD$ is also greater than the internal and opposite angle $\angle ABC$.



[It may be useful to make reference to the above figure, in which the point E bisects the lines $[AC]$ and $[BF]$. Consider the relationships between the lengths and angles of the triangles $\triangle AEB$ and $\triangle FEC$. You may use, without proof, standard congruence rules satisfied by triangles in the plane.]

Euclid, Book I, Proposition 17

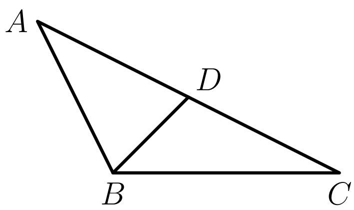
Prove that, in any triangle, the sum of two angles is less than two right angles



[It may be useful to make reference to the above figure. You may use without proof the result of Euclid's *Elements*, Book I, Proposition 13, which ensures that the sum of two supplementary angles is equal to two right angles.]

Euclid, Book I, Proposition 18

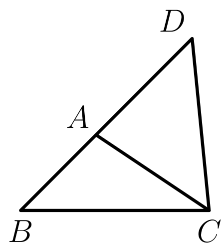
Prove that if, in a triangle $\triangle ABC$, the side $[AC]$ is greater than the side $[AB]$, then the angle $\angle ABC$ opposite the greater side $[AC]$ is greater than the angle $\angle ACB$ opposite the lesser side $[AB]$.



[It may be useful to make reference to the above figure, in which the line segments $[AB]$ and $[AD]$ are equal to one another.]

Euclid, Book I, Proposition 20

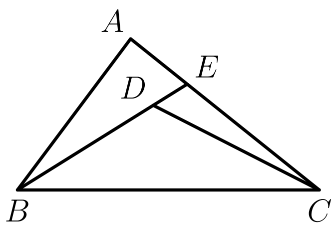
Prove that, in a triangle $\triangle ABC$, the sum of the two sides $[AB]$ and $[AC]$ is greater than the base $[BC]$.



[It may be useful to make reference to the above figure, in which B, A and D are collinear and the line segments $[AD]$ and $[AC]$ are equal to one another.]

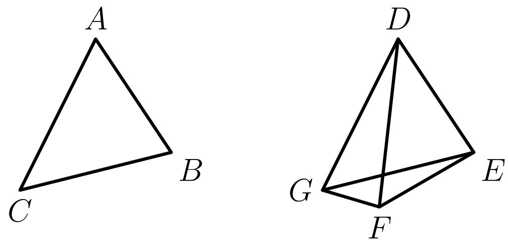
Euclid, Book I, Proposition 21

Let D be a point in the interior of a triangle $\triangle ABC$ (see the figure below). Prove that the sum of $[BD]$ and $[DC]$ is less than the sum of $[BA]$ and $[AC]$. Prove also that the angle $\angle BDC$ is greater than the angle $\angle BAC$.



Euclid, Book I, Proposition 24, in a particular case

Let $\triangle ABC$ and $\triangle DEF$ be triangles for which the sides $[AB]$ and $[DE]$ are equal to one another, the sides $[AC]$ and $[DF]$ are equal to one another, and for which the angle $\angle EDF$ is less than the angle $\angle BAC$. Also let G be a point determined so that the triangle $\triangle DEG$ is congruent to $\triangle ABC$. Suppose that the points D and F lie on opposite sides of $[EG]$, as depicted in the figure below. Prove that the side $[EF]$ is less than $[BC]$.



Euclid, Book I, Proposition 26 (ASA Congruence Rule)

Let $\triangle ABC$ and $\triangle DEF$ be triangles. Suppose that the side $[BC]$ of the first triangle is equal to the side $[EF]$ of the second triangle, and that the angles of the first triangle at the vertices B and C are respectively equal to the angles of the second triangle at E and F. Prove that the triangles $\angle ABC$ and $\angle DEF$ are congruent.

[You may use, without proof, the *SAS* Congruence Rule.]

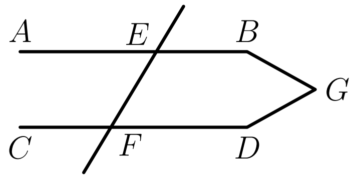
Euclid, Book I, Proposition 26 (SAA Congruence Rule)

Let $\triangle ABC$ and $\triangle DEF$ be triangles. Suppose that the side $[AB]$ of the first triangle is equal to the side $[DE]$ of the second triangle, and that the angles of the first triangle at the vertices B and C are respectively equal to the angles of the second triangle at E and F. Prove that the triangles $\angle ABC$ and $\angle DEF$ are congruent.

[You may use, without proof, the *SAS* Congruence Rule. You may also use, without proof, the result of Proposition 16 of Book I of Euclid's *Elements*, which asserts that an exterior angle of a triangle is greater than either of the interior and opposite angles.]

Euclid, Book I, Proposition 27

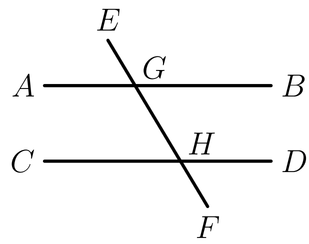
Let a straight line $[EF]$ fall on two straight lines $[AB]$ and $[CD]$, as depicted in the figure below



Suppose that the alternate angles $\angle AEF$ and $\angle EFD$ are equal to one another. Prove that the lines $[AB]$ and $[CD]$ are then parallel.

Euclid, Book I, Proposition 28

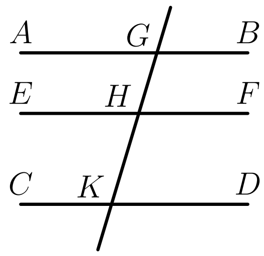
Let a straight line $[EF]$ fall on two straight lines $[AB]$ and $[CD]$, as depicted in the figure below, intersecting $[AB]$ at G and intersecting $[CD]$ at H.



Prove that the lines AB and CD are parallel, in the case where the exterior angle $\angle EGB$ is equal to the interior and opposite angle $\angle GHD$. Prove also that the lines AB and CD are parallel, in the case where the sum of the interior angles $\angle BGH$ and $\angle GHD$ is equal to two right angles.

Euclid, Book I, Proposition 30

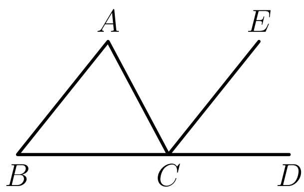
Using the results of Propositions 27, 28 and 29 of Book I of Euclid's *Elements*, prove that if straight lines AB and CD are both parallel to a straight line EF then they are parallel to one another.



[It may be useful to make reference to the above figure.]

Euclid, Book I, Proposition 32

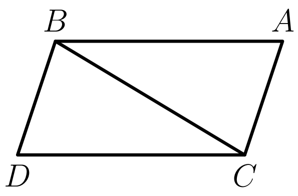
Let $\triangle ABC$ be a triangle, and let the side $[BC]$ be produced beyond C to D. Using the result of Proposition 29 of Euclid, prove that the exterior angle $\angle ACD$ is equal to the sum of the two interior and opposite angles $\angle CAB$ and $\angle ABC$. Prove also that the sum of the interior angles of the triangle $\triangle ABC$ at its vertices A, B and C is equal to two right angles.



[It may be useful to make reference to the above figure, in which the line CE is parallel to the edge $[AB]$ of the triangle $\triangle ABC$.]

Euclid, Book I, Proposition 33

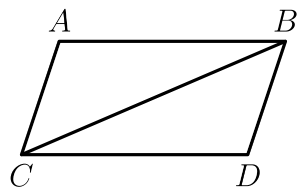
Consider the quadrilateral $ABDC$ depicted below:



Suppose that the sides $[AB]$ and $[CD]$ are parallel and equal to one another. Prove that the sides $[AC]$ and $[BD]$ are also parallel and equal to one another.

Euclid, Book I, Proposition 34

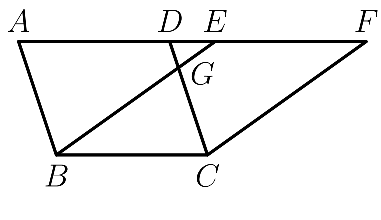
Consider the quadrilateral $ABDC$ depicted below:



Suppose that the sides $[AB]$ and $[CD]$ are parallel to one another and that the sides $[AC]$ and $[BD]$ are also parallel to one another. Prove that the sides $[AB]$ and $[CD]$ are equal to one another. Prove also sides $[AC]$ and $[BD]$ are equal to one another. Prove also that the angle $\angle CAB$ is equal to the angle $\angle BDC$ and that the angle $\angle ABD$ is equal to the angle $\angle DCA$.

Euclid, Book I, Proposition 35

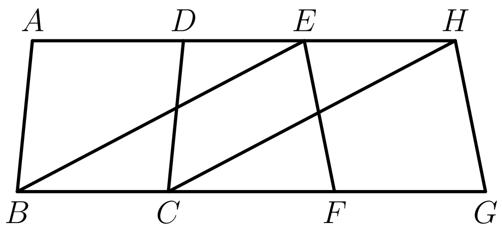
Consider the configuration depicted below, in which the lines BC and AF are parallel and $ABCD$ and $EBCF$ are parallelograms on the same base $[BC]$ and between the same parallels BC and AF.



Prove that the parallelograms $ABCD$ and $EBCF$ are equal (in area) to each other.

Euclid, Book I, Proposition 36

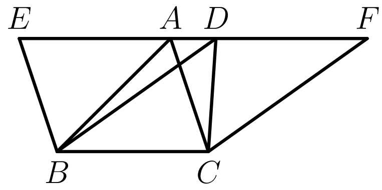
Consider the configuration depicted below, in which the lines BG and AH are parallel and $ABCD$ and $EFGH$ are parallelograms on equal bases $[BC]$ and $[FG]$ and between the same parallels BG and AH.



Prove that the parallelograms $ABCD$ and $EFGH$ are equal (in area) to each other.

Euclid, Book I, Proposition 37

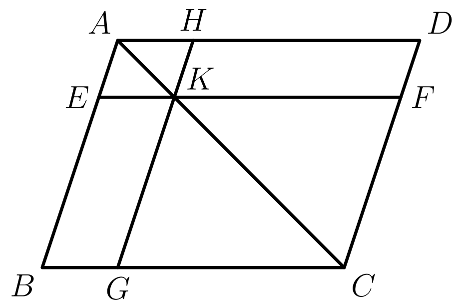
Let $\triangle ABC$ and $\triangle DBC$ be triangles on the same base $[BC]$ and in the same parallels AB, BC [see figure]. Explain why triangles $\angle ABC$ is equal to the triangle $\triangle DBC$.



[It may be useful to make reference to the above figure. You may use, without proof, the result that if two parallelograms, such as the parallelograms $EABC$ and $DFBC$ in the figure, are on the same base and between the same parallels then those paralellograms are equal in area. You may also use, without proof, the result that any diagonal of a parallelogram bisects the parallelogram, dividing it into two triangles of equal area.]

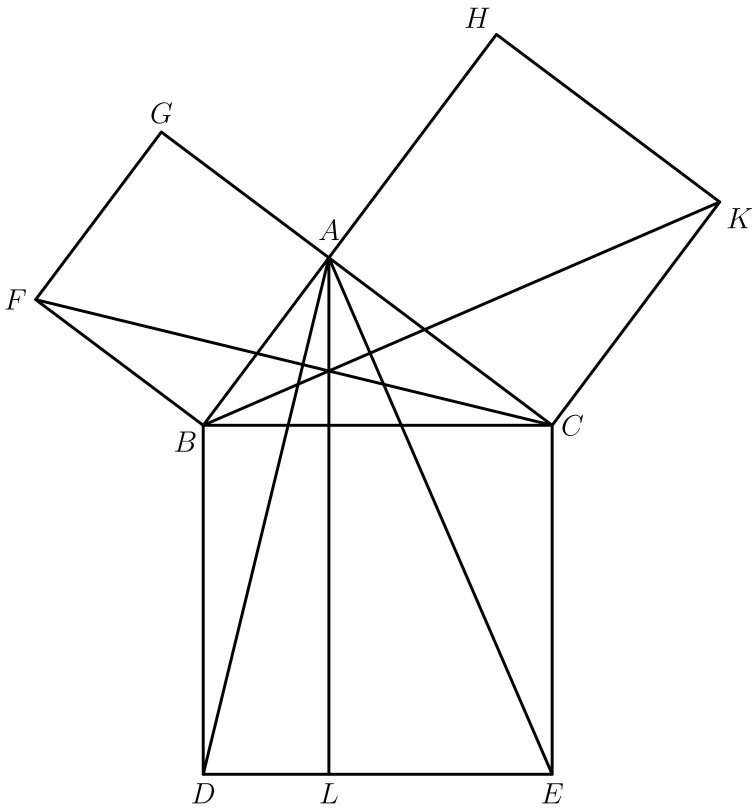
Euclid, Book I, Proposition 43

Consider the configuration depicted below, in which $ABCD$ is a parallelogram and $[EF]$ and $[GH]$ are lines from one edge to an opposite edge, as shown, which intersect at a point K. Prove that the "complements" $BGKE$ and $KFDH$ are equal (in area) to one another.



Euclid, Book I, Proposition 47, Pythagoras's Theorem

Prove that, if a triangle $\triangle ABC$ has a right angle at the vertex A then the square on the hypotenuse $[AC]$ is equal to the sum of the squares on the other two sides $[AB]$ and $[AC]$. Your proof should be within the theory presented in Book I of Euclid's *Elements*, and thus should be based on the postulates, common notions, unstated assumptions and preceding propositions of that book, and not on results from later books or theories of synthetic geometry distinct from the framework presented in Book I of Euclid's *Elements*. In particular, the proof should be based around the concept of equality of area, for geometric figures, and should not require the theory of proportion and/or similar triangles.



[It may be useful to make reference to the above figure, in which $BCDE$ is the square on the hypotenuse and $ABFG$ and $ACKH$ are the squares on the sides $[AB]$ and $[AC]$ respectively.]

Euclid, Book I, Proposition 48, Converse of Pythagoras's Theorem

Prove that, if the square on the longest side $[AC]$ of a triangle $\triangle ABC$ is equal to the sum of the squares on the other two sides, then the triangle is a right-angled triangle. [You may use, without proof, Pythagoras's Theorem.]