Course MA1S11: Michaelmas Term 2016.
Tutorial 7: Sample
November 21–25, 2016

Results that may be useful.

Quadratic Polynomials

Let a, b and c be real or complex numbers, where $a \neq 0$. The roots of the quadratic polynomial $ax^2 + bx + c$ are given by the quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Differentiation by Rule

Let x be a real variable, taking values in a subset D of the real numbers, and let y, u and v dependent variables, expressible as functions of the independent variable x, that are differentiable with respect to x. Then the following results are valid:—

(i) if $y = c$, where c is a real constant, then $\dfrac{dy}{dx} = 0$;

(ii) if $y = cu$, where c is a real constant, then $\dfrac{dy}{dx} = c\dfrac{du}{dx}$;

(iii) if $y = u + v$ then $\dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$;

(iv) if $y = x^q$, where q is a rational number, then $\dfrac{dy}{dx} = qx^{q-1}$;

(v) *(Product Rule)* if $y = uv$ then $\dfrac{dy}{dx} = u\,\dfrac{dv}{dx} + v\,\dfrac{du}{dx}$;

(vi) *(Quotient Rule)* if $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \dfrac{v\,\dfrac{du}{dx} - u\,\dfrac{dv}{dx}}{v^2}$;

(vii) *(Chain Rule)* if y is expressible as a differentiable function of u, where u in turn is expressible as a differentiable function of x, then $\dfrac{dy}{dx} = \dfrac{dy}{du}\,\dfrac{du}{dx}$.

Problem 1

Determine the derivatives of the following functions, using standard rules for differentiation such as the Product Rule, Quotient Rule and Chain Rule where appropriate.

(a)	$\dfrac{7-2x}{x^2-6x+20}$	
(b)	$x^2\sqrt[5]{6+x^4}$	
(c)	$\dfrac{x^4}{\sqrt{x^2+11}}$	

It is recommended that you show your working on the sheets at the end of the tutorial handout, but answers should be entered into the boxes above.

Problem 2

Let $f\colon [2,6] \to \mathbb{R}$ be defined so that

$$f(x) = 15x^{\frac{11}{5}} - 275x^{\frac{6}{5}} + 3465x^{\frac{1}{5}}.$$

for all real numbers x satisfying $2 \leq x \leq 9$. Determine the values of x in the interval $[2,9]$ for which $f'(x) = 0$, and determine also whether those values are local maxima or local minima.

Additional Work