

Course MA1S11: Michaelmas Term 2016.

Tutorial 3: Sample

October 18–21, 2016

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Results that may be useful.

Quadratic Polynomials and related Functions

Let a , b and c be real or complex numbers, where $a \neq 0$. The roots of the quadratic polynomial $ax^2 + bx + c$ are given by the quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Moreover if $a = 1$ then the sum of the roots is $-b$ and the product of the roots is c .

Let a and c be real constants, where $a > 0$ and $c > 0$, and let $F: (0, +\infty) \rightarrow \mathbb{R}$ be the function on the set of positive real numbers defined such that

$$F(x) = ax + \frac{c}{x}$$

for all positive real numbers x . A real number y satisfies the equation $y = F(x)$ for some positive real number x if and only if $ax^2 - yx + c = 0$. The real numbers x (if any) that satisfy the equation $y = F(x)$ can therefore be determined from the quadratic formula. The minimum value of $F(x)$ on $(0, +\infty)$ is $2\sqrt{ac}$, and this value is attained when $x = \sqrt{c/a}$.

Notation for intervals

Let a and b be real numbers, where $a \leq b$. Then

$$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}, \quad (a, b] = \{x \in \mathbb{R} \mid a < x \leq b\},$$

$$[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}, \quad (a, b) = \{x \in \mathbb{R} \mid a < x < b\},$$

$$(-\infty, a] = \{x \in \mathbb{R} \mid x \leq a\}, \quad (-\infty, a) = \{x \in \mathbb{R} \mid x < a\},$$

$$[a, +\infty) = \{x \in \mathbb{R} \mid x \geq a\}, \quad (a, +\infty) = \{x \in \mathbb{R} \mid x > a\}.$$

Functions

A function $f: X \rightarrow Y$ maps elements of a set X to elements of a set Y . The set X is the *domain* of the function $f: X \rightarrow Y$, and the set Y is the *codomain* of this function. The function $f: X \rightarrow Y$ is *injective* if it maps distinct elements of domain X to distinct elements of the codomain Y . Thus $f: X \rightarrow Y$ is injective if and only if

$$u, v \in X \text{ and } f(u) = f(v) \Rightarrow u = v.$$

The *range* of the function $f: X \rightarrow Y$ is the set $f(X)$, where

$$f(X) = \{f(x) | x \in X\}.$$

The function $f: X \rightarrow Y$ is *surjective* if $f(X) = Y$. The function $f: X \rightarrow Y$ is *bijective* if it is both injective and surjective. An *inverse* $g: Y \rightarrow X$ for the function $f: X \rightarrow Y$ is a function $g: Y \rightarrow X$ with the properties that $g(f(x)) = x$ for all $x \in X$ and $f(g(y)) = y$ for all $y \in Y$. A function $f: X \rightarrow Y$ has a well-defined inverse $g: Y \rightarrow X$ if and only if it is bijective.

Problem 1

- (a) Let $f: (1, 54] \rightarrow \mathbb{R}$ be defined such that $f(x) = 3x + \frac{108}{x}$ for all $x \in (1, 54]$.

- (i) Is the function $f: (1, 54] \rightarrow \mathbb{R}$ injective? Tick the correct box below:—

☐ *Yes*

☐ *No*

- (ii) What is the range of the function $f: (1, 54] \rightarrow \mathbb{R}$? Write your answer in the box below as an interval in one of the forms $[a, b]$, $[a, b)$, $(a, b]$, (a, b) as appropriate, where a and b are suitably-chosen real numbers satisfying $a < b$.

- (b) Let $g: (2, 6] \rightarrow \mathbb{R}$ be defined such that $g(x) = 3x + \frac{108}{x}$ for all $x \in (2, 6]$.

- (i) Is the function $g: (2, 6] \rightarrow \mathbb{R}$ injective? Tick the correct box below:—

☐ *Yes*

☐ *No*

- (ii) What is the range of the function $g: (2, 6] \rightarrow \mathbb{R}$? Write your answer in the box below as an interval in one of the forms $[a, b]$, $[a, b)$, $(a, b]$, (a, b) as appropriate, where a and b are suitably-chosen real numbers satisfying $a < b$.

Problem 2

- (a) Consider the function $f: [0, 4] \rightarrow [0, 64]$ defined such that

$$f(x) = \begin{cases} 37 + 9x & \text{if } 0 \leq x \leq 3; \\ 64 - x^3 & \text{if } 3 < x \leq 4. \end{cases}$$

for all $x \in [0, 4]$.

- (i) Is the function $f: [0, 4] \rightarrow [0, 64]$ injective? Tick the correct box below:—

☐ *Yes*

☐ *No*

If you claim that this function is not injective, give a reason for your answer in not more than three sentences in the following box:

- (ii) Is the function $f: [0, 4] \rightarrow [0, 64]$ surjective? Tick the correct box below:—

☐ *Yes*

☐ *No*

If you claim that this function is not surjective, give a reason for

your answer in not more than three sentences in the following box:

- (iii) Is the function $f: [0, 4] \rightarrow [0, 64]$ bijective? Tick the correct box below:—

Yes

No

-
- (b) Consider the function $g: [0, 4] \rightarrow [0, 64]$ defined such that

$$g(x) = \begin{cases} 37 + 9x & \text{if } 0 \leq x < 3; \\ 64 - x^3 & \text{if } 3 \leq x \leq 4. \end{cases}$$

for all $x \in [0, 4]$.

- (i) Is the function $g: [0, 4] \rightarrow [0, 64]$ injective? Tick the correct box below:—

Yes

No

If you claim that this function is not injective, give a reason for your answer in not more than three sentences in the following box:

- (ii) Is the function $g: [0, 4] \rightarrow [0, 64)$ surjective? Tick the correct box below:—

Yes

No

If you claim that this function is not surjective, give a reason for your answer in not more than three sentences in the following box:

- (iii) Is the function $g: [0, 4] \rightarrow [0, 64)$ bijective? Tick the correct box below:—

Yes

No

-
- (c) Consider the function $h: [0, 4] \rightarrow [0, 64]$ defined such that

$$h(x) = \begin{cases} 64 - 8x & \text{if } 0 \leq x < 3; \\ x^3 - 27 & \text{if } 3 \leq x \leq 4. \end{cases}$$

for all $x \in [0, 4]$.

- (i) Is the function $h: [0, 4] \rightarrow [0, 64]$ injective? Tick the correct box below:—

Yes

No

If you claim that this function is not injective, give a reason for

your answer in not more than three sentences in the following box:

- (ii) Is the function $h: [0, 4] \rightarrow [0, 64]$ surjective? Tick the correct box below:—

Yes

No

If you claim that this function is not surjective, give a reason for your answer in not more than three sentences in the following box:

- (iii) Is the function $h: [0, 4] \rightarrow [0, 64]$ bijective? Tick the correct box below:—

Yes

No