Course MA1S11: Michaelmas Term 2016.

Tutorial 2: Sample

October 11–14, 2016

Solutions

Results that may be useful.

The following statement provides a summary of the laws of indices proved in lectures that are applicable to powers of real numbers.

> *The "laws of indices" encapsulated in the formulae $a^{p+q} = a^p a^q$, $a^{pq} = (a^p)^q$ and $(ab)^p = a^p b^p$ are valid in the following situations:—*
>
> - *when a and b are real numbers and p and q are non-negative integers;*
> - *when a and b are non-zero real numbers and p and q are integers;*
> - *when a and b are positive real numbers and p and q are rational numbers.*

Let a, b and c be real or complex numbers, where $a \neq 0$. The roots of the quadratic polynomial $ax^2 + bx + c$ are given by the quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Moreover if $a = 1$ then the sum of the roots is $-b$ and the product of the roots is c.

Trigonometric formulae

Let θ and φ denote angles measured in radians. The sine, cosine and tangent functions $\sin\theta$, $\cos\theta$ and $\tan\theta$ satisfy the identities

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1,\\
\tan\theta &= \frac{\sin\theta}{\cos\theta},\\
\sin(\theta+\varphi) &= \sin\theta\,\cos\varphi + \cos\theta\,\sin\varphi,\\
\cos(\theta+\varphi) &= \cos\theta\,\cos\varphi - \sin\theta\,\sin\varphi,\\
\sin(-\theta) &= -\sin\theta,\\
\cos(-\theta) &= \cos\theta.
\end{aligned}$$

Problem 1

Rational numbers p and q can be determined so that

$$\frac{x^3}{\sqrt[5]{x^2 + \frac{1}{\sqrt[3]{x^4}}}} = \frac{x^p}{\sqrt[5]{1 + x^q}}.$$

for all positive real numbers x. Determine values of p and q, and write them in the boxes below:

EITHER

$$p = \boxed{\frac{13}{5}} \qquad q = \boxed{-\frac{10}{3}}$$

Then write down the resultant expression (with specific values substituted for p and q) in the following box:

$$\frac{x^3}{\sqrt[5]{x^2 + \frac{1}{\sqrt[3]{x^4}}}} = \boxed{\frac{x^{\frac{13}{5}}}{\sqrt[3]{1 + x^{-\frac{10}{3}}}}}.$$

OR ELSE

$$p = \boxed{\frac{49}{15}} \qquad q = \boxed{\frac{10}{3}}$$

Then write down the resultant expression (with specific values substituted for p and q) in the following box:

$$\frac{x^3}{\sqrt[5]{x^2 + \frac{1}{\sqrt[3]{x^4}}}} = \boxed{\frac{x^{\frac{49}{15}}}{\sqrt[3]{1 + x^{\frac{10}{3}}}}}.$$

Problem 2

In parts (a), (b), (c) below, roots of quadratic polynomials may be real or complex. If the polynomial has a repeated root, please write "repeated" in the box for the second root.

(a) Write down the roots of the quadratic polynomial $x^2 - 6x + 9$ in the boxes below:

3	repeated

.

(b) Write down the roots of the quadratic polynomial $x^2 + 10x + 24$ in the boxes below:

-4	-6

.

(c) Write down the roots of the quadratic polynomial $x^2 + 8x + 14$ in the boxes below:

$-4 + \sqrt{2}$	$-4 - \sqrt{2}$

.

(d) Write down the roots of the quadratic polynomial $x^2 - 4x + 20$ in the boxes below:

$2 + 4i$	$2 - 4i$

.

Problem 3

(a) Find all *real numbers* x that are roots of the polynomial

$$x^{14} - 133x^{11} + 1000x^8.$$

(The algorithm for polynomial division is not needed. Hint: $1000 = 8 \times 125 = 2^3 \times 5^3$.) List these real numbers in the box below:—

0, 2, 5

(b) Find all *positive real numbers* x that satisfy

$$x^{\frac{5}{8}} - 7x^{\frac{1}{2}} + 10x^{\frac{3}{8}} = 0.$$

List these real numbers in the box below:—

256, 390625

Problem 4

Let θ satisfy $0 < \theta < \frac{\pi}{2}$, so that $\sin\theta > 0$ and $\cos\theta > 0$. Each of the quantities labelled (a), (b), (c) and (d) is equal to one of the formulae labelled (i), (ii), (iii) and (iv). Fill in the table below by matching up one of the formulae labelled (a)–(d) with one of the formulae labelled (i)–(iv).

(a) $\sqrt{1-\cos^2\theta}$, (b) $1+\tan^2\theta$, (c) $\cos^2\theta - \sin^2\theta$, (d) $\dfrac{\cos^2\theta}{1+\sin\theta}$,

(i) $\dfrac{1}{\cos^2\theta}$, (ii) $1-\sin\theta$, (iii) $\cos 2\theta$, (iv) $\sin\theta$.

(a)	(b)	(c)	(d)
(iv)	(i)	(iii)	(ii)