# Course MA1S11: Michaelmas Term 2016.

## Tutorial 1: Sample

## October 4–7, 2016

## Solutions

**Problem 1**

*Let intervals be defined as follows:—*

$$I = (2,8), \quad J = [1,15], \quad K = [6,12), \quad L = [11,17].$$

*Also let sets A to H be defined as follows:—*

$$\begin{aligned} A &= [1,17], \\ B &= (2,17], \\ C &= [6,8), \\ D &= [6,11), \\ E &= [8,11), \\ F &= (2,6) \cup [11,17], \\ G &= (2,6) \cup [12,17], \\ H &= (2,8) \cup [11,17]. \end{aligned}$$

*Each of the sets tabulated below is equal to one of the sets A–H. In each row of the table below, enter the set from A–H that is equal to the specified set*

| | |
|---|---|
| $I \cup L$ | $H$ |
| $K \setminus L$ | $D$ |
| $(I \cup L) \setminus K$ | $G$ |
| $J \cup (L \setminus K)$ | $A$ |
| $(I \cup L) \cup (K \setminus L)$ | $B$ |
| $(I \cup L) \cap (K \setminus L)$ | $C$ |
| $(I \cup L) \setminus (K \setminus L)$ | $F$ |
| $(K \setminus L) \setminus (I \cup L)$ | $E$ |

$$\begin{aligned} A &= [1,17], \\ B &= (2,17], \\ C &= [6,8), \\ D &= [6,11), \\ E &= [8,11), \\ F &= (2,6) \cup [11,17], \\ G &= (2,6) \cup [12,17], \\ H &= (2,8) \cup [11,17]. \end{aligned}$$

$$
\begin{aligned}
I \cup L &= (2,8) \cup [11,17] = H, \\
K \setminus L &= [6,11) = D, \\
(I \cup L) \setminus K &= (2,6) \cup [12,17] = G, \\
J \cup (L \setminus K) &= [1,17] = A, \\
(I \cup L) \cup (K \setminus L) &= (2,17] = B, \\
(I \cup L) \cap (K \setminus L) &= [6,8) = C, \\
(I \cup L) \setminus (K \setminus L) &= (2,6) \cup [11,17] = F, \\
(K \setminus L) \setminus (I \cup L) &= [8,11) = E.
\end{aligned}
$$

**Problem 2**: prove the identity

$$\sum_{j=1}^{n} \frac{4}{(3j+4)(3j+1)} = \frac{n}{3n+4}.$$

by induction on $n$.

**Proof by Induction.** If $n = 1$ then

$$\sum_{j=1}^{n} \frac{4}{(3j+4)(3j+1)} = \frac{4}{7 \times 4} = \frac{4}{28} = \frac{1}{7} = \frac{n}{3n+4}.$$

Thus the result holds when $n = 1$. Suppose the result holds when $n = k$, so that

$$\sum_{j=1}^{k} \frac{4}{(3j+4)(3j+1)} = \frac{k}{3k+4}.$$

Then

$$\begin{aligned}
\sum_{j=1}^{k+1} \frac{4}{(3j+4)(3j+1)} &= \sum_{j=1}^{k} \frac{4}{(3j+4)(3j+1)} + \frac{4}{(3(k+1)+4)(3k+4)} \\
&= \frac{k}{3k+4} + \frac{4}{(3(k+1)+4)(3k+4)} \\
&= \frac{k(3k+7)}{(3k+7)(3k+4)} + \frac{4}{(3k+7)(3k+4)} \\
&= \frac{3k^2+7k+4}{(3k+7)(3k+4)} \\
&= \frac{(k+1)(3k+4)}{(3k+7)(3k+4)} \\
&= \frac{k+1}{3k+7} \\
&= \frac{k+1}{3(k+1)+4}.
\end{aligned}$$

Thus if the result holds when $n = k$ then it also holds when $n = k + 1$. It follows from the Principle of Mathematical Induction that the result holds for all natural numbers $n$.