Course 2BA1: Michaelmas Term 2007.

Sample Problems concerning Induction

1. Prove by induction on n that

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = 2 - \frac{n+2}{n+1}$$

   for all natural numbers n.

2. Prove by induction on n that the product $1 \times 3 \times \cdots \times (2n-1)$ of the first n odd natural numbers is equal to $\dfrac{(2n)!}{2^n n!}$.

3. Prove by induction on n that $(3n)! > 2^{6n-4}$ for all natural numbers n.

4. Prove by induction on n that

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1},$$

   for all natural numbers n, where

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)}.$$

5. Prove by induction on n that $n! > 3^{n-2}$ for all natural numbers n satisfying $n \geq 3$ (where $n!$ denotes the product of all natural numbers from 1 to n inclusive).

6. Prove by induction on n that

$$\sum_{i=1}^{n} 4^{i-1} i(i+1) = \frac{1}{27}((9n^2 + 3n + 2)4^n - 2)$$

   for all natural numbers n.

7. Prove by induction on n that $(n!)^2 \geq 2^{2n-2}$ for all natural numbers n (where $n!$ denotes the product of all natural numbers from 1 to n inclusive).

8. Prove by induction on n that

$$\sum_{i=1}^{n} \frac{2i+1}{i^2(i+1)^2} = \frac{n^2+2n}{(n+1)^2}.$$

9. Prove by induction on n that $(3n)! \geq \frac{1}{20} \times 120^n$ for all natural numbers n (where $n!$ denotes the product of all natural numbers from 1 to n inclusive).

10. Prove by induction on n that

$$\sum_{i=1}^{n} (i^3 + i) > \frac{1}{4}(n^4 + n)$$

for all natural numbers n.

11. Use the Method of Mathematical Induction to prove that

$$\sum_{k=1}^{n} \frac{1}{(k+2)(k+3)(k+4)} = \frac{1}{24} - \frac{1}{2(n+3)(n+4)}.$$

for all positive integers n.