Course 2BA1: Hilary Term 2008.

Sample Problems concerning Formal Languages

1. Devise a context-free grammar to generate the language over the alphabet $\{0, 1\}$ consisting of the strings

$$01, \ 0011, \ 000111, \ 00001111, \ldots$$

(i.e., consisting of m zeros, for some non-negative integer m, followed by m ones). You should specify the nonterminals of the grammar, the start symbol and the productions of the grammar.

2. (a) Devise a regular grammar to generate the language over the alphabet $\{a, (,), 0, 1\}$ consisting of all strings such as `a(001)` and `a(1001010)` in which the initial substring `a(` is followed by a non-empty string of binary digits, which is followed by the character `)`.

(b) Devise a finite state acceptor that accepts (i.e., determines) the language described in (a). You should specify the states of the machine, the start state, the finishing state(s), and the transition table that defines the machine.

3. Consider the context free-grammar over the alphabet $\{x, y, (,)\}$ with nonterminals $\langle S\rangle$ and $\langle A\rangle$, start symbol $\langle S\rangle$ and productions

$$\langle S\rangle \to \langle A\rangle\langle A\rangle, \quad \langle A\rangle \to (\langle S\rangle), \quad \langle A\rangle \to x, \quad \langle A\rangle \to y.$$

Show that the string $(x(yx))y$ belongs to the language over the alphabet $\{x, y, (,)\}$ generated by this grammar. Does the string $(x(xy$ belong to this language? [Briefly justify your answer.]

4. (a) Devise a regular grammar to generate the language over the alphabet $\{x, y, z\}$ consisting of all strings

$$xyz, xyzxyz, xyzxyzxyz, xyzxyzxyzxyz, \ldots$$

that are the concatenation of n copies of the string xyz for some positive integer n.

(b) Devise a finite state acceptor that accepts (i.e., determines) the language described in (a). You should specify the states of the machine, the start state, the finishing state(s), and the transition table that defines the machine.

5. (a) Describe the formal language over the alphabet $\{0, 1\}$ generated by the context-free grammar whose only non-terminal is $\langle S \rangle$, whose start symbol is $\langle S \rangle$ and whose productions are the following:

$$\begin{aligned} \langle S \rangle &\rightarrow 0 \\ \langle S \rangle &\rightarrow 0\langle S \rangle \\ \langle S \rangle &\rightarrow \langle S \rangle 1 \end{aligned}$$

Is this context-free grammar a regular grammar?

(b) Devise a regular grammar to generate the language over the alphabet $\{a, b, c\}$ consisting of all finite strings, such as *ab*, *aabbb*, *aaaaab*, *abc*, *aabbbc*, that consist of one or more occurrences of the character a, followed by one or more occurrences of the character b, optionally followed by a single occurrence of the character c.

(c) Give the definition of a finite state acceptor that accepts (or determines) the language described in (b). You should specify the states of the machine, the start state, the finishing state or states, and the transition table that defines the machine.