Course 2BA1: Academic Year 2000–1.

Assignment V.

To be handed in by Wednesday 4th May, 2001.
Please include both name and student number on any work handed in.

1. Let $f\colon \mathbb{R} \to \mathbb{R}$ be the periodic function with period 2π given for real numbers x satisfying $-\pi \le x \le \pi$ by the formula

$$f(x) = \begin{cases} 1 - \dfrac{2|x|}{\pi} & \text{if } -\tfrac{1}{2}\pi \le x \le \tfrac{1}{2}\pi; \\[2mm] 0 & \text{if } -\pi \le x \le -\tfrac{1}{2}\pi \text{ or } \tfrac{1}{2}\pi \le x \le \pi. \end{cases}$$

(Here $|x|$, the absolute value of x, is defined by $|x| = x$ if $x \ge 0$, and $|x| = -x$ if $x < 0$.) The function f can be expanded as a Fourier series of the form

$$f(x) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx.$$

(The terms involving $\sin nx$ are zero since the given function is even.) Find the coefficients a_n of this series, and hence write down the Fourier series for the function f.