

Course 2BA1: Hilary Term 2003.

Assignment IV.

To be handed in by Wednesday 5th March, 2003.

Please include both name and student number on any work handed in.

1. Let c be a fixed positive integer, and let \otimes denote the binary operation on the set \mathbb{Z} of integers defined by the formula

$$x \otimes y = xy + c(x + y) + c^2 - c$$

for all integers x, y and z .

- (a) Is (\mathbb{Z}, \otimes) a semigroup? [Justify your answer.]
 - (b) Is (\mathbb{Z}, \otimes) a monoid? If so, what is its identity element?
 - (c) Which of the elements of \mathbb{Z} are invertible? Is (\mathbb{Z}, \otimes) a group?
2. Let q and r be the quaternions defined by $q = \frac{1}{2} + \frac{1}{2}i + \frac{1}{2}j + \frac{1}{2}k$ and $r = i - j$. Calculate $qr\bar{q}$ (where \bar{q} denotes the conjugate of the quaternion q).
 3. Construct a regular grammar that generates the language L over the alphabet $\{0, 1\}$, where

$$L = \{1, 1000, 1000000, 1000000000, \dots\},$$

so that a string of binary digits belongs to L if and only if it consists of the digit 1 followed by a string of $3n$ zeroes, for some non-negative integer n . You should specify your formal grammar in Backus-Naur form.