Course 2BA1: Hilary Term 2002.

Assignment IV.

To be handed in by Wednesday 6th February, 2002.
Please include both name and student number on any work handed in.

1. Let $S = \{0, 1, 2\}$, and let $\wedge$ be the binary operation on S defined such that $x \wedge y$ is the maximum of x and y for all $x, y \in S$ (so that $0 \wedge 1 = 1$, $1 \wedge 1 = 1$, $2 \wedge 0 = 2$ etc.).

   (a) Is $(S, \wedge)$ a semigroup? [Justify your answer.]

   (b) Is $(S, \wedge)$ a monoid? If so, what is its identity element?

   (c) Which of the elements of S are invertible? Is $(S, \wedge)$ a group?

2. Let $(A, *)$ be a monoid with identity element e, let x and q be invertible elements of A, let x^{-1} and q^{-1} be the inverses of x and q respectively, and let

   $$y = (q * x) * q^{-1}.$$

   Prove that y is invertible, with inverse given by

   $$y^{-1} = (q * x^{-1}) * q^{-1}.$$

   [You should not appeal directly to the General Associative Law, for products involving four or more elements, but you should make successive applications of the Associative Law for products involving three elements, and use the definitions and basic properties of identity elements and inverses.]

3. Let q and r be the quaternions defined by $q = 2j + k$, $r = j + 2k$. Calculate the quaternion products qr and rq.