Course 2BA1: Hilary Term 2009.

Assignment III.

To be handed in by Wednesday 15th April, 2009.
Please include both name and student number on any work handed in.

1. Describe the formal language over the alphabet $\{0, 1, 2\}$ generated by the context-free grammar whose only non-terminal is $\langle S \rangle$, whose start symbol is $\langle S \rangle$ and whose productions are the following:

$$\begin{aligned} \langle S \rangle &\rightarrow 0 \\ \langle S \rangle &\rightarrow 1\langle S \rangle 2 \end{aligned}$$

   (i.e., describe the structure of the binary strings generated by the grammar). Is this context-free grammar a regular grammar?

2. (a) Give the specification of a finite state acceptor that accepts the language over the alphabet $\{0, 1\}$ whose words consist of a single 1, followed a string of 0's, where the number of such 0's is congruent to 3 modulo 5. (In particular you should specify the set of states, the starting state, the finishing states, and the transition table that determines the transition function of the finite state acceptor.)

   (b) Devise a regular grammar to generate the language specified in (a).

3. Evaluate the products qr and rq of the quaternions q and r, where $q = 35 + i - 4j + 3k$ and $r = 4 + 2i + j$.

4. Use the Euclidean Algorithm to find the greatest common divisor d of the integers 53298, 11844 and 6909, and find integers a, b and c such that $d = 53298a + 11844b + 6909c$.

5. Find an integer x with the properties that $x \equiv 7 (\mathrm{mod}\, 13)$, $x \equiv 3 (\mathrm{mod}\, 7)$ and $x \equiv 3 (\mathrm{mod}\, 5)$.