

Course 2BA1: Trinity Term 2007.

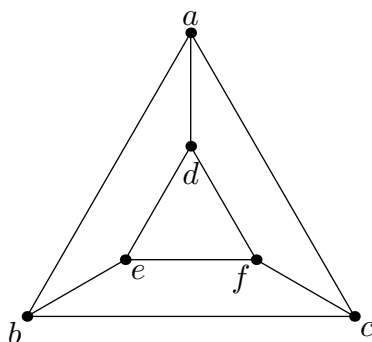
Assignment III.

To be handed in by Friday 4th May, 2007.

Please include both name and student number on any work handed in.

- (a) Let A be the set $\mathbb{C} \times \mathbb{C}$ consisting of all ordered pairs (z, w) , where z and w are complex numbers. Let \times denote the binary operation on A defined by $(z, w) \times (u, v) = (zu - wv, zv + wu)$ for all complex numbers z, w, u and v . Prove that (A, \times) is a monoid. What is its identity element? Prove that an element (z, w) of A is invertible if and only if $z^2 + w^2 \neq 0$.

(b) Let (\mathbb{C}, \times) be the monoid consisting of the set of complex numbers with the usual operation of multiplication, and let $f: \mathbb{C} \rightarrow A$ be the function from \mathbb{C} to A which sends the complex number $x + iy$ to the ordered pair (x, y) for all real numbers x and y . Is the function f a homomorphism from (\mathbb{C}, \times) to (A, \times) ? Is this function an isomorphism?



- Answer the following questions concerning the graph with vertices a, b, c, d, e and f pictured above. [Justify all your answers.]
 - Is the graph complete?
 - Is the graph regular?
 - Is the graph connected?
 - Does the graph have an Eulerian circuit?

- (e) Does the graph have a Hamiltonian circuit?
- (f) Give an example of a spanning tree for the graph, specifying the vertices and edges of the spanning tree.
- (g) Given an example of an isomorphism between the graph pictured above and that pictured below. (You should specify the isomorphism as a function between the sets $\{a, b, c, d, e, f\}$ and $\{u, v, w, x, y, z\}$ of vertices of the two graphs.)

