Course 2BA1: Hilary Term 2006.
Assignment III.

To be handed in by Wednesday 19th April, 2006.
Please include both name and student number on any work handed in.

1. Let $\oplus$ be the binary operation on the set $\mathbb{Z}$ of integers defined by $x \oplus y = x + y - 7$. Prove that $(\mathbb{Z}, \oplus)$ is a monoid. What is the identity element of this monoid? Is this monoid a group?

2. Let q and r be the quaternions given by $q = 2i - j$ and $r = i - k$. Calculate the quaternion products $q \times r$ and $r \times q$ (expressing $q \times r$ and $r \times q$ in the form $w + xi + yj + zk$ for appropriate real numbers w, x, y and z).

3. (a) Describe the formal language over the alphabet $\{0, 1\}$ generated by the context-free grammar whose only non-terminal is $\langle S \rangle$, whose start symbol is $\langle S \rangle$ and whose productions are the following:

$$\begin{aligned}
\langle S \rangle &\rightarrow 0 \\
\langle S \rangle &\rightarrow 0\langle S \rangle \\
\langle S \rangle &\rightarrow \langle S \rangle 1
\end{aligned}$$

Is this context-free grammar a regular grammar?

(b) Devise a regular grammar to generate the language over the alphabet $\{a, b, c\}$ consisting of all finite strings, such as ab, $aabbb$, $aaaaab$, abc, $aabbbc$, that consist of one or more occurrences of the character a, followed by one or more occurrences of the character b, optionally followed by a single occurrence of the character c.

(c) Give the definition of a finite state acceptor that accepts (or determines) the language described in (b). You should specify the states of the machine, the start state, the finishing state or states, and the transition table that defines the machine.

4. Obtain the general solution of the following ordinary differential equation:
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = x \cos x.$$