Course 2BA1: Hilary Term 2005.

Assignment III.

To be handed in by Wednesday 13th April, 2005.
Please include both name and student number on any work handed in.

1. Let $\mathbb{R}^2$ denote the set of all ordered pairs (x, y), where x and y are real numbers, and let $\otimes_1$ and $\otimes_2$ denote the binary operations on $\mathbb{R}^2$ defined by the formulae

$$\begin{aligned}(x, y) \otimes_1 (u, v) &= (xv + yu,\ yv + xu),\\ (x, y) \otimes_2 (u, v) &= (xv + yu,\ yv - xu)\end{aligned}$$

   for all real numbers x, y, u, v? Is $(\mathbb{R}^2, \otimes_1)$ a monoid? Is $(\mathbb{R}^2, \otimes_2)$ a monoid?

2. Let q and r be the quaternions given by $q = i - j$ and $r = 2 + i - k$. Calculate the quaternion products $q \times r$ and $r \times q$ (expressing $q \times r$ and $r \times q$ in the form $w + xi + yj + zk$ for appropriate real numbers w, x, y and z).

3. Consider the context free-grammar over the alphabet $\{x, y, (,)\}$ with nonterminals $\langle S\rangle$ and $\langle A\rangle$, start symbol $\langle S\rangle$ and productions

$$\langle S\rangle \to \langle A\rangle\langle A\rangle, \quad \langle A\rangle \to (\langle S\rangle), \quad \langle A\rangle \to x, \quad \langle A\rangle \to y.$$

   Show that the string $(x(yx))y$ belongs to the language over the alphabet $\{x, y, (,)\}$ generated by this grammar. Does the string $(x(xy$ belong to this language? [Briefly justify your answer.]

4. (a) Devise a regular grammar to generate the language over the alphabet $\{x, y, z\}$ consisting of all strings

$$xyz, xyzxyz, xyzxyzxyz, xyzxyzxyzxyz, \ldots$$

   that are the concatenation of n copies of the string xyz for some positive integer n.

   (b) Devise a finite state acceptor that accepts (i.e., determines) the language described in (a). You should specify the states of the machine, the start state, the finishing state(s), and the transition table that defines the machine.