Course 2BA1: Hilary Term 2004.

Assignment III.

To be handed in by Wednesday 7th April, 2004.
Please include both name and student number on any work handed in.

1. Let X denote the set

$$\{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$$

of odd integers, and let $\#$ denote the binary operation on X defined by $x \# y = \frac{1}{2}(xy - x - y + 3)$ for all odd integers x and y.

(a) Is $(X, \#)$ a monoid? If so, what is its identity element?

(a) Is $(X, \#)$ a group?

[Briefly justify your answers.]

2. Let q and r be the quaternions given by $q = 1 - i - j$ and $r = 2i + j - k$. Calculate the quaternion products $q \times r$ and $r \times q$ (expressing $q \times r$ and $r \times q$ in the form $w + xi + yj + zk$ for appropriate real numbers w, x, y and z).

3. Devise a context-free grammar to generate the language over the alphabet $\{0, 1\}$ consisting of the strings

$$01,\ 0011,\ 000111,\ 00001111, \ldots$$

(i.e., consisting of m zeros, for some non-negative integer m, followed by m ones). You should specify the nonterminals of the grammar, the start symbol and the productions of the grammar.

4. (a) Devise a regular grammar to generate the language over the alphabet $\{a, (,), 0, 1\}$ consisting of all strings such as `a(001)` and `a(1001010)` in which the initial substring `a(` is followed by a non-empty string of binary digits, which is followed by the character `)`.

(b) Devise a finite state acceptor that accepts (i.e., determines) the language described in (a). You should specify the states of the machine, the start state, the finishing state(s), and the transition table that defines the machine.