Course 2BA1: Academic Year 2000–1.

Assignment III.

To be handed in by Friday 2nd February, 2001.
Please include both name and student number on any work handed in.

1. Let E denote the set

$$\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$$

of even integers, let $+$ and $\times$ denote the usual arithmetic operations of addition and multiplication respectively, and let $\#$ denote the binary operation on E defined by $x \# y = \frac{1}{2}xy$ for all even integers x and y.

(a) Is $(E, +)$ a monoid?

(b) Is $(E, \times)$ a monoid?

(c) Is $(E, \#)$ a monoid?

[Briefly justify your answers.]

2. Let q and r be the quaternions given by $q = 1 - i$ and $r = 2i - j - k$. Calculate the quaternion products $q \times r$ and $r \times q$ (expressing $q \times r$ and $r \times q$ in the form $w + xi + yj + zk$ for appropriate real numbers w, x, y and z).

3. Let $(A, *)$ be a monoid, let s be an invertible element of A, and let $f: A \to A$ be the function from A to itself defined by $f(x) = (s * x) * s^{-1}$ for all elements x of A (where s^{-1} denotes the inverse of s).

(a) Prove that the function f is a homomorphism.

(b) Let $g: A \to A$ be the function defined by $g(x) = (s^{-1} * x) * s$ for all elements x of A. Prove that the function g is the inverse of the function f.

(c) Is the function f an isomorphism?