# Course 2BA1: Michaelmas Term 2008.

# Assignment II.

**To be handed in by Wednesday 14th January, 2009.**
**Please include both name and student number on any work handed in.**

1. Let $f\colon [1,4] \to [-1,8]$ and $g\colon [1,4] \to [0,4]$ be the functions defined such that $f(x) = x^3 - 6x^2 + 12x - 8$ and $g(x) = x^2 - 4x + 4$ for all $x \in [1,4]$. Which (if any) of the functions $f$ and $g$ are injective? Which are surjective? Which are invertible?

2. Consider a graph with vertices $a$, $b$, $c$, $d$, $e$ and $f$ and edges $a\,b$, $a\,c$, $b\,d$, $b\,e$, $b\,f$, $c\,d$, $d\,e$ and $d\,f$.

   (a) Draw a diagram showing the vertices and edges of this graph.

   (b) Is this graph regular?

   (c) Is this graph complete?

   (d) Does this graph have an Eulerian circuit? If so, give an example.

   (e) Give an example of a spanning tree for this graph.

   (f) There are exactly three simple circuits in this graph that commence with the edge $b\,f$. Write down all three of these circuits (specifying the vertices of the circuit in the order in which they are passed through). Does this graph have a Hamiltonian circuit?

   (g) Give an example of an isomorphism between this graph and the graph with vertices Consider a graph with vertices $p$, $q$, $r$, $s$, $t$ and $u$ and edges $p\,q$, $p\,r$, $p\,s$, $p\,t$, $q\,r$, $r\,s$, $r\,u$ and $t\,u$.

   [Briefly justify all your answers above.]

3. Let $X$ be the set $\{x \in \mathbb{R} : x > 2\}$ consisting of all real numbers greater than 2, and let $\otimes$ be the binary operation on $X$ defined such that

$$x \otimes y = \frac{xy - 4}{x + y - 4}$$

   for all $x, y \in X$. Prove that $(X, \otimes)$ is a semigroup. Is $(X, \otimes)$ a monoid? [Justify your answer.]