

# Course 2BA1: Michaelmas Term 2007.

## Assignment II.

To be handed in by Wednesday 16th January, 2008.

Please include both name and student number on any work handed in.

1. Let  $A$  be the set consisting of all ordered pairs  $(x, y)$ , where  $x$  and  $y$  are integers, and let  $\otimes$  denote the binary operation on  $A$  defined by

$$(x_1, y_1) \otimes (x_2, y_2) = (x_1x_2 + 2y_1y_2, x_1y_2 + y_1x_2).$$

for all integers  $x_1, x_2, y_1$  and  $y_2$ . Prove that  $(A, \otimes)$  is a monoid. What is its identity element?

2. Consider the graph  $(V, E)$ , where

$$V = \{a, b, c, d, e, f\},$$

$$E = \{ab, ac, ad, ae, bc, bd, bf, ce, cf, de, df, ef\}.$$

Answer the following questions concerning this graph. [Justify all your answers.]

- (a) Is the graph complete?
- (b) Is the graph regular?
- (c) Is the graph connected?
- (d) Does the graph have an Eulerian circuit?
- (e) Does the graph have a Hamiltonian circuit?
- (f) Give an example of a spanning tree for the graph, specifying the vertices and edges of the spanning tree.
- (g) Given an example of an isomorphism between the graph pictured above and the graph  $(V', E')$ , where

$$V' = \{p, q, r, s, t, u\},$$

$$E' = \{pq, ps, pt, pu, qr, qt, qu, rs, rt, ru, st, su\}.$$